# TIME DELAY BASEDUNKNOWN INPUT OBSERVER DESIGN FOR NETWORK CONTROL SYSTEM

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**ABSTRACT**

*Many modern control systems use communication networks to exchange information among system's components. Due to insertion of communication network these systems are often subjected to disturbances, perturbation etc. This paper deals with the observer design problem for the network control systems with unknown inputs. In this paper, the existing approach with one delay in feedback systemhas been extended by inserting two communication networks with individual respective delays in observation and control channels. Network effect is manifested in terms of time-delayin signals exchanged between components.A numerical example is also presented and simulated using MATLAB/SIMULINK to demonstrate the approach.*

*Index Terms: Network Control System, Observer, Simulink, Time Delay,Unknown Input.*

## 1. INTRODUCTION

A control system in which its components such as plant, sensors, actuators are spatially distributed is called Network Control System (NCS). In such systems, communication takes place via a band-limited communication network. One of the most prominent feature of the NCS is that exchange of information between system's components takes place in form of information packets. NCSs has been an area of extensive research over the past few decades due to its increased flexibility, lower costs, simple installation and easy maintenance etc. On the other hand, there has been some serious issues due to the use of communication network such limited bandwidth, network induced delay, packet dropout. These problems must be look into as these can have adverse effect on system stability. The typical NCS structure is shown in Fig. 1.

### 1.1 Literature review

The roots of observer design can be traced back to late 1960s when Luenberger first designed observer in 1966 [3]. Since then, different types of observers have been developed which are being used in various engineering applications. As the large scale systems are more susceptible to high disturbance and faults, the study of UIOs become more pivotal [2]. Most of the UIOsproposed in literature are for non-networked systems [2]. Bhattacharya [4] provided the solution to the observer design problem for systems subjected to unknown inputs or

disturbances.One simple method was proposed by Darouach et al. [5] for designing the full order observer, the necessary and sufficient conditions for this observer were given. Kobayashi and Nakamizo [6] suggested an approach based on Silverman's inverse method. On the other hand Hui and Zak [7] proposed a projection operator approach.

Recently an UIO was designed by Taha et al. [2] for network control system, stability guarantee bounds for the delay and perturbation induced by network were also established.
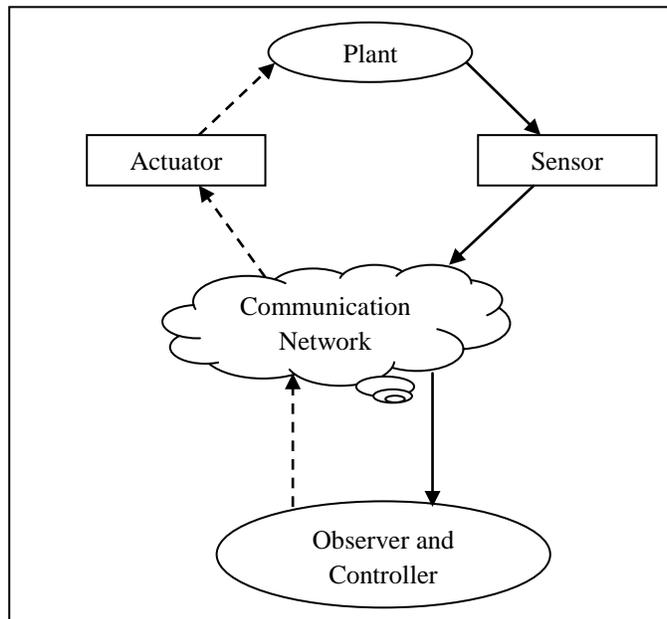


**Fig 1 Typical Network Control System**

## II SYSTEM MODELLING

Proposed architecture for a networked unknown input observer scheme is shown in Fig 2. The input to the observer block are theknown input $u_1$ and delayed version of the plant's output, that is $\hat{y}$. We have assumed that these two quantities are known to the observer block. The innovation block is different for different observers [2]. Estimate of plant state, that is $\hat{x}_m$ is the output of the observer. Reference input ($V_{ref}$) and $\hat{x}_m$ acts as inputs to the controller. The unknown input for the plant is taken as $u_2$ that can be any non-linearity or any unknown plant disturbance.

### 2.1 Review of unknown input observer for non-networked systems:

Zak & Hui [1] introduced a projection operator approach based on which observer is designed in this paper. We have considered the Linear-Time invariant (LTI) class of systems. The dynamics of plant is given as: $\dot{x}_m = A_m x_m + B_m^{(1)} u_1 + B_m^{(2)} u_2$ (1)      where $A_m \in \square^{n \times n}$, $B_m^1 \in \square^{n \times m_1}$, $B_m^2 \in \square^{n \times m_2}$, $x_m$ is plant state, $u_1$ is known input, $u_2$ is unknown input, $y$ is output respectively. It is assumed that $A_m, B_m^1, B_m^2$ are all

known system parameters. For the non-networked system, the dynamics of unknown input observer (UIO) as

presented in [1] are: $\dot{z} = (I - MC_m)(A_m z + A_m My + B_m^{(1)} u_1 + L(y - C_m z - C_m My))$
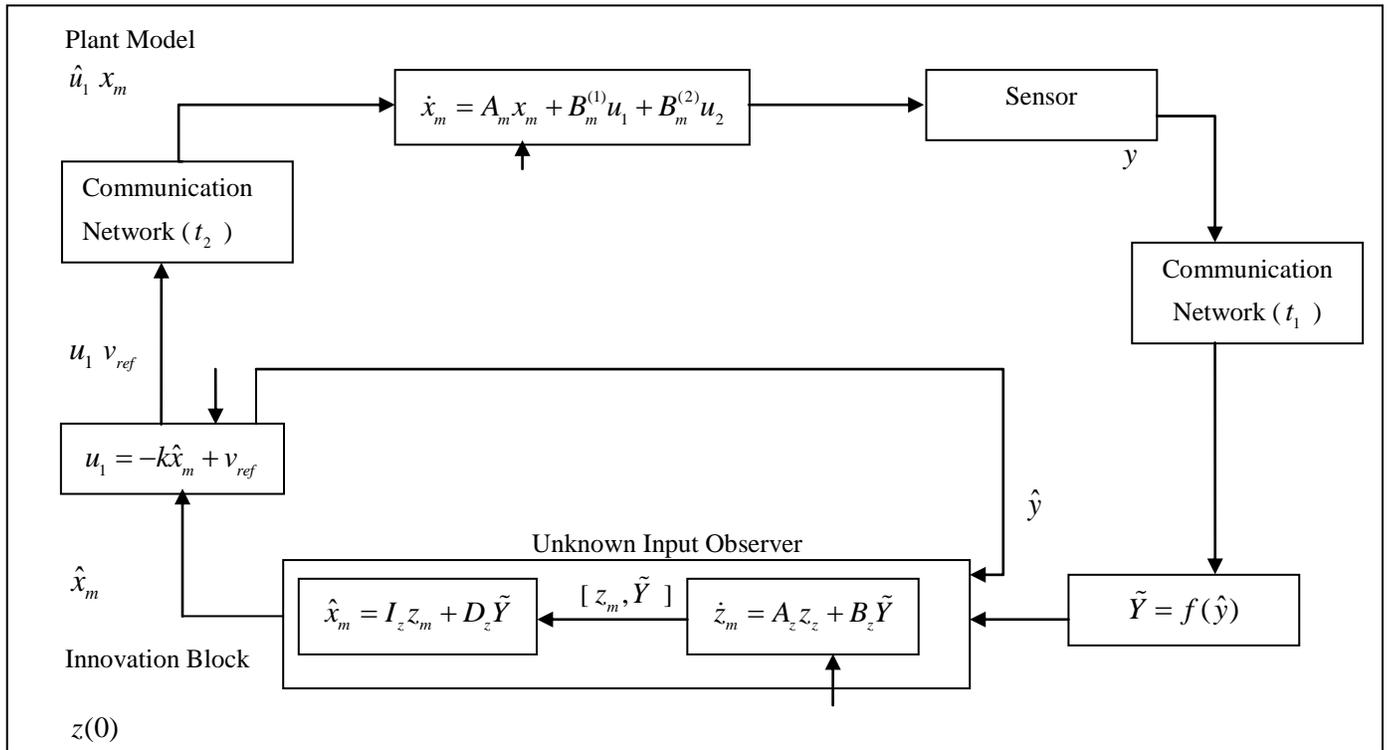
(2)  $\hat{x}_m = z + My$  (3)



**Fig 2. Networked UIO Architecture**

where $M \in \square^{n \times p}$ is chosen such that

$$(I - MC_m)B_m^{(2)} = 0 \qquad\qquad\qquad\qquad (4)$$

and $L$ is the additional gain to improve the convergence rate [2]. The initial conditions for the observer for the above

dynamical system are $z(0) = (I - MC_m)\hat{x}(0)$ \qquad (5)

The designed observer for the non-networked system converges estimation error to zeros as t→∞, under the

assumption that the pair $(C_m, A_m)$ is detectable[1]. In order to analyse the effect of communication network on state

estimation, the dynamics of the observer by taking $x_c = z$ can be written as follows:

$$\dot{x}_c = (I - MC_m)(A_m x_c + A_m My + B_m^{(1)} u_1 + L(y - C_m x_c - C_m My)) \quad (6)$$

$\dot{x}_c = A_c x_c + B_c^{(1)} y + B_c^{(2)} u_1$ (7) where

$$A_c = (I - MC_m)(A_m - LC_m), \ B_c^{(1)} = (I - MC_m)(A_m M + L - LC_m M), \ B_c^{(2)} = (I - MC_m)B_m^{(1)}$$ The

dynamics of the UIO will get changed due the presence of the communication network between plant and observer

and can be written as follows:
$$\dot{x}_c = A_c x_c + B_c^{(1)} \hat{y} + B_c^{(2)} u_1 \quad (8) \quad \hat{x}_m = x_c + M\hat{y} \ (9)$$ we

assume state feedback control is used:
$$u_1 = -k\hat{x}_m = -k(x_c + M\hat{y})$$

$$(10)$$

## III NETWORK EFFECT AS PURE TIME DELAY

### 3.1 Closed Loop Dynamics

As can be seen in figure (2), we have inserted two communication networks one between unknown input observer

(UIO) and its inputs, and another one between controller and plant due to which plant gets the delayed input. In this

$$\hat{y} = y(t - t_1)$$

section, we model the communication network effect as pure time-delay. So, it is assumed that $\hat{u} = u(t - t_2)$

$$(11)$$

where $t_1$ and $t_2$ are the time-delays occurring due to network effect. Rewriting the dynamics of the observer and

controller in the distinct form of NCS observer/controller while assuming that the innovation function of the

observer is embedded in UIO dynamics :

$$\dot{x}_c(t) = A_c x_c(t) + B_c^{(1)} y(t - t_1) + B_c^{(2)} u_1(t) \ (12) \quad u_1(t) = C_c x_c(t) + D_c y(t - t_1) \ (13)$$ where

$C_c = -k$ and $D_c = -kM$ State dynamics of plant and controller can be written as:

$$\dot{x}_m(t) = A_m x_m(t) + B_m^{(1)} u_1(t - t_2) + B_m^{(2)} u_2(t) \ (14)$$

$$= A_m x_m(t) + B_m^{(1)} C_c x_c(t - t_2) + B_m^{(1)} D_c C_m x_m(t - (t_1 + t_2)) + B_m^{(2)} u_2(t)$$

$$\dot{x}_c(t) = A_c x_c(t) + B_c^{(1)} y(t - t_1) + B_c^{(2)} u_1(t)$$

$$(15) = A_c x_c(t) + B_c^{(1)} C_m x_m(t - t_1) + B_c^{(2)} C_c x_c(t) + B_c^{(2)} D_c C_m x_m(t - t_1)$$

Combining the above two equations to find $\dot{x}(t)$
$$\dot{x}(t) = \begin{bmatrix} \dot{x}_m(t) \\ \dot{x}_c(t) \end{bmatrix} \ (16)$$

By simplifying (16) we get,

$$\dot{x}(t) = \Gamma_0 x(t) + \Gamma_1 x(t - t_1) + \Gamma_2 x(t - t_2) + \Gamma_3 x(t - (t_1 + t_2)) + \Gamma_4 u_2(t) \ (17)$$

where

$$\Gamma_0 = \begin{bmatrix} A_m & 0 \\ 0 & A_c + B_c^{(2)} C_c \end{bmatrix}, \ \Gamma_1 = \begin{bmatrix} 0 & 0 \\ B_c^{(1)} C_m + B_c^{(2)} D_c C_m & 0 \end{bmatrix}, \ \Gamma_2 = \begin{bmatrix} 0 & B_m^{(1)} C_c \\ 0 & 0 \end{bmatrix},$$

$$\Gamma_3 = \begin{bmatrix} B_m^{(1)} D_c C_m & 0 \\ 0 & 0 \end{bmatrix}, \ \Gamma_4 = \begin{bmatrix} B_m^{(2)} \\ 0 \end{bmatrix}$$

### 3.2 NCS and Approximation of Time-Delay

From [2] we have,

$$\dot{x}_m(t - \tau) = 0, \dot{x}_c(t - \tau) = 0$$

Taking the second derivative of x(t) and substituting the above approximation, we have,

$$\ddot{x}(t) = \Gamma_0 \dot{x}(t) + \Gamma_4 \dot{u}_2(t) \ (18)$$

Using the following taylor series expansion for $x(t\text{-}a)$:

$$x(t - a) = \sum_{n=0}^{\infty} (-1)^n \frac{a^n}{n!} x^{(n)}(t) \ (19)$$

Neglecting the higher order terms, we get

$$x(t - t_1) = x(t) - t_1 \dot{x}(t) + \frac{t_1^2}{2} \ddot{x}(t) \ (20) \quad x(t - t_2) = x(t) - t_2 \dot{x}(t) + \frac{t_2^2}{2} \ddot{x}(t) \ (21)$$

$$x(t - t_3) = x(t) - t_3 \dot{x}(t) + \frac{t_3^2}{2} \ddot{x}(t) \ (22) \text{ where } t_3 = t_1 + t_2$$

Substituting (20),(21),(22) in (17) we get

$$\dot{x}(t) = \Gamma_0 x(t) + \Gamma_1 [x(t) - t_1 \dot{x}(t) + \frac{t_1^2}{2} \ddot{x}(t)] + \Gamma_2 [x(t) - t_2 \dot{x}(t) + \frac{t_2^2}{2} \ddot{x}(t)] + \Gamma_3 [x(t) - t_3 \dot{x}(t) + \frac{t_3^2}{2} \ddot{x}(t)] + \Gamma_4 u_2(t) \ (23)$$

Substituting (18) in above equation

$$\dot{x}(t) = \Gamma_0 x(t) + \Gamma_1 [x(t) - t_1 \dot{x}(t) + \frac{t_1^2}{2} (\Gamma_0 \dot{x}(t) + \Gamma_4 \dot{u}_2(t))] + \Gamma_2 [x(t) - t_2 \dot{x}(t) + \frac{t_2^2}{2} (\Gamma_0 \dot{x}(t) + \Gamma_4 \dot{u}_2(t))]$$

$$(23) + \Gamma_3 [x(t) - t_3 \dot{x}(t) + \frac{t_3^2}{2} (\Gamma_0 \dot{x}(t) + \Gamma_4 \dot{u}_2(t))] + \Gamma_4 u_2(t)$$

$$(24)$$

Rearranging the above terms, we get $\dot{x}(t) = \Phi_0 x(t) + \Phi_1 \dot{u}_2(t) + \Phi_2 u_2(t) \ (25)$

where

$$\Phi_0 = \Theta(\Gamma_0 + \Gamma_1 + \Gamma_2)$$

$$\Phi_1 = \Theta(\frac{t_1^2}{2}\Gamma_4\Gamma_1 + \frac{t_2^2}{2}\Gamma_4\Gamma_2 + \frac{t_3^2}{2}\Gamma_4\Gamma_3) \;\; \Theta = (I - (-t_1\Gamma_1 + \frac{t_1^2}{2}\Gamma_0\Gamma_1 - t_2\Gamma_2 + \frac{t_2^2}{2}\Gamma_0\Gamma_2 - t_3\Gamma_3 + \frac{t_3^2}{2}\Gamma_0\Gamma_3))^{-1}$$

$$\Phi_2 = \Theta\Gamma_4$$

### 3.3 Time-Delay Based UIO Design for Networked System

In this section, controller and observer is designed in order to minimize the effect of multiple delay terms as well as of the uknown input from the dynamics of the closed loop system.

We know,

$$\Gamma_3 = \begin{bmatrix} B_m^{(1)} D_c C_m & 0 \\ 0 & 0 \end{bmatrix}, \; \Gamma_4 = \begin{bmatrix} B_m^{(2)} \\ 0 \end{bmatrix} \text{ and } \Gamma_3\Gamma_4 = \begin{bmatrix} B_m^{(1)} D_c C_m B_m^{(2)} & 0 \\ 0 & 0 \end{bmatrix}$$

The observer is to be designed in such a way that effect of multiple delay terms and unknown input is nullified. So, this means $\Gamma_3\Gamma_4$ should result equal to zero i.e, $\Gamma_3\Gamma_4 = 0$ $B_m^{(1)} D_c C_m B_m^{(2)} = 0$ or $B_m^{(1)} D_c = 0$ $(26)$ Substituting the value of $D_c = -kM$ in the above equation, we get $-B_m^{(1)}kM = 0$ and we know that the lone condition for the existence of UIO for the non-networked systems as in [1]: $(I - MC_m)B_m^{(2)} = 0$ $(27)$ So, to design the observer we have the following equations to solve: $-B_m^{(1)}kM = 0$ $(28)$ $(I - MC_m)B_m^{(2)} = 0$

$$(29)$$ As we don't have much control over $k$ so $M$ is the variable to be solved for the design of the Networked UIO [2].

Now multiplying equation (29) by a non-singular matrix $H \in \Box^{m_2 \times p}$ and then adding it in equation (28), we get $-B_m^{(1)}kM + B_m^{(2)}H - MC_m B_m^{(2)}H = 0_{n \times p}$ $(30)$

we get the quadratic equation in terms of $M$ i.e, Networked UIO

Let $R = B_m^{(1)}k \in \Box^{n \times n}$, then above equation can be represented design variable.

as $-RM + B_m^{(2)}H - MC_m B_m^{(2)}H = 0_{n \times p}$ $(31)$

$$MA + BM + C = 0_{n \times p} \;\; (32)$$

Above equation can be written as follows:

Hence the Networked UIO can be designed by solving equation (32).


### IV UNKNOWN INPUT OBSERVER DESIGN EXAMPLE FOR A NETWORKED SYSTEM

In this section, we demonstrate the example for the proposed Networked UIO design following the algorithm 1 and algorithm 2 given in [2]. We assume that the given system has one known input, one unknown input and one output.

$$A_m = \begin{bmatrix} -5 & 3 & 0 \\ 4 & -10 & 4 \\ 0 & 0 & -4 \end{bmatrix}, B_m^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, B_m^{(2)} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, C_m = \begin{bmatrix} 2 & 4 & -1 \end{bmatrix}$$

1)     Networked UIO existence:

rank($C_m B_m^{(2)}$)=1; rank($B_m^{(2)}$) =1

As both ranks are equal to 1, so the UIO exist.

2)     Compute the design matrix $M$ :

Following the algorithm 1 given in [2], we get     $M = \begin{bmatrix} 0.1812 & 0.1667 & 0.0291 \end{bmatrix}^t$

3)     Compute $P_1$ :

$$P_1 = I_n - M C_m = \begin{bmatrix} 0.6376 & -0.7248 & 0.1812 \\ -0.3334 & 0.3332 & 0.1667 \\ -0.0582 & -0.1164 & 1.0291 \end{bmatrix}$$

4)     Find $Q$ :

$$Q = \begin{bmatrix} 0.8833 & -0.7309 & 0.5279 \\ -0.4618 & -0.6724 & -0.0520 \\ -0.0806 & -0.1174 & 0.8477 \end{bmatrix}$$

5)     Compute $\tilde{A}_m$ and $\tilde{C}_m$ :

$$\tilde{A}_m = \begin{bmatrix} \tilde{A}_m^{(11)} & \tilde{A}_m^{(12)} \\ \tilde{A}_m^{(21)} & \tilde{A}_m^{(22)} \end{bmatrix} = \begin{bmatrix} -9.8171 & -1.3766 & -4.5565 \\ -4.8066 & -3.9982 & -5.4268 \\ -1.2187 & -0.1307 & -5.1847 \end{bmatrix} \tilde{C}_m = \begin{bmatrix} \tilde{C}_m^{(1)} & \tilde{C}_m^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & -4.0338 & 0 \end{bmatrix}$$

6)     Compute eigen values of $\tilde{A}_m^{(11)}$ :

eigen($\tilde{A}_m^{(11)}$)= $[-10.7912, -3.0241]$

7)     Since the eigen values of $\tilde{A}_m^{(11)}$ are negative which means that the matrix is stable, then ($\tilde{A}_m^{(11)}, \tilde{C}_m^{(1)}$) is detectable.

8)     Design gain L:

$L$= place $(A_m^t, C_m^t, \begin{bmatrix} -10 & -11 & -12 \end{bmatrix})^t = \begin{bmatrix} 1.4348 \\ 4.6087 \\ 7.3043 \end{bmatrix}$

9) Compute NCS parameters i.e, $A_c$, $B_c^{(1)}$, $B_c^{(2)}$ :

$$A_c = P_1(A_m - LC_m) = \begin{bmatrix} -3.8832 & 13.5689 & -4.7260 \\ -1.5500 & -13.4318 & 2.9409 \\ -13.9685 & -26.5984 & 2.3149 \end{bmatrix}, B_c^{(1)} = P_1(A_m M + L - LC_m M) = \begin{bmatrix} 0.3188 \\ -0.1595 \\ -0.0007 \end{bmatrix},$$

$$B_c^{(2)} = P_1 B_m^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

UIO observer for the networked system can be written as follows:

$$\dot{x}_c = A_c x_c + B_c^{(1)} \hat{y} + B_c^{(2)} u_1$$

$$\hat{x}_m = x_c + M\hat{y}$$

We assume the same initial condition of plant state and unknown input observer state as used in [2] i.e,

$$x_m(0) = \begin{bmatrix} 0.356 \\ -0.0422 \\ -0.719 \end{bmatrix} \text{ and } x_c(0) = \begin{bmatrix} -6.66 \\ -1.66 \\ -6.66 \end{bmatrix},$$

and the unknown input is taken as $u_2(t) = 0.5\sin(t)$. Fig. (3) and Fig (4) shows the estimation of plant states.

## V SIMULATION RESULTS

### 5.1 For Non-Networked System ($t_1 = t_2 = 0$)

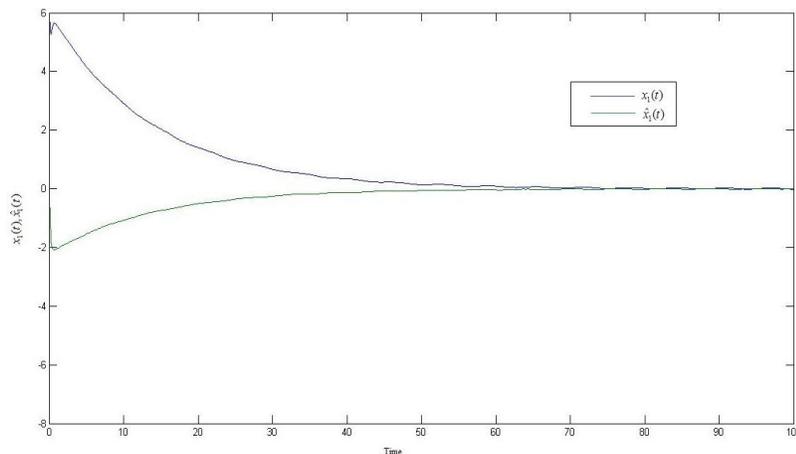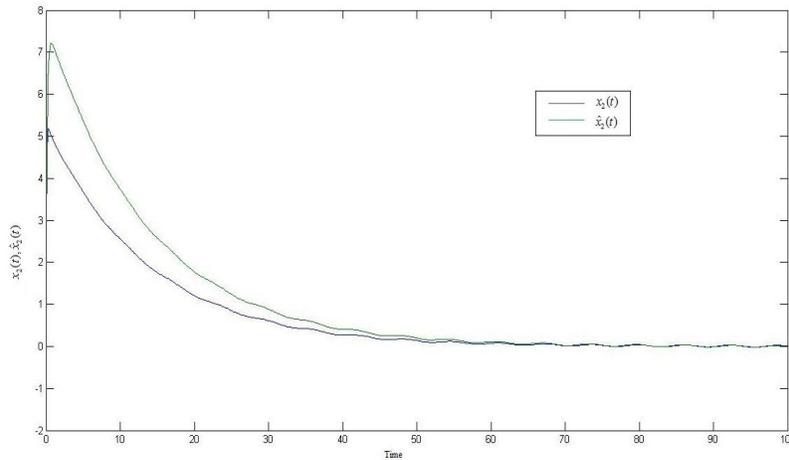Fig. 3 shows the estimation of plant states for non-networked system with unknown inputs.
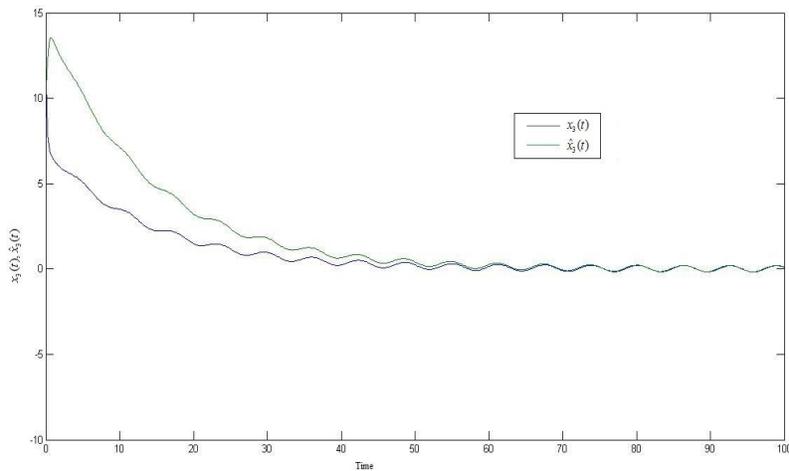


Fig 3(a)

Fig. 3(b)



Fig. 3(c)

**Fig. 3 Simulation Results of Estimation of States for Non-Networked UIO**

## 5.2  For Networked System ($t_1 \neq 0, t_2 \neq 0$)

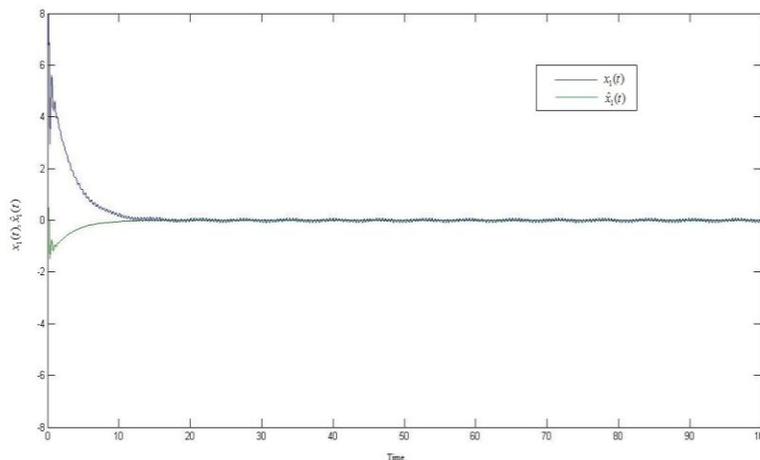Fig. 4 shows the estimation of plant states for networked control system.
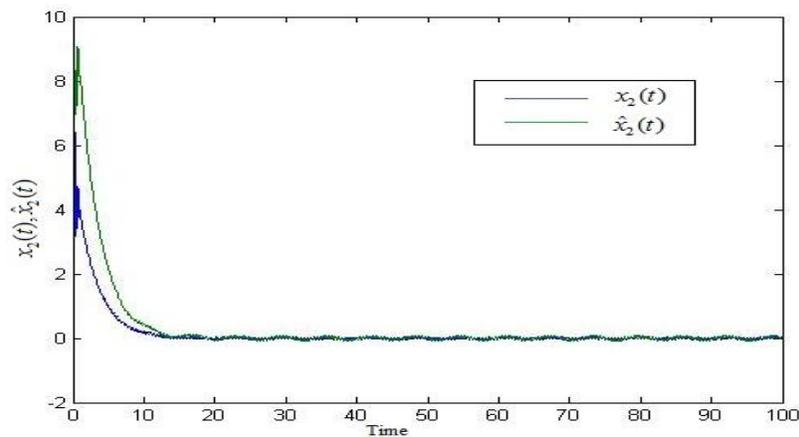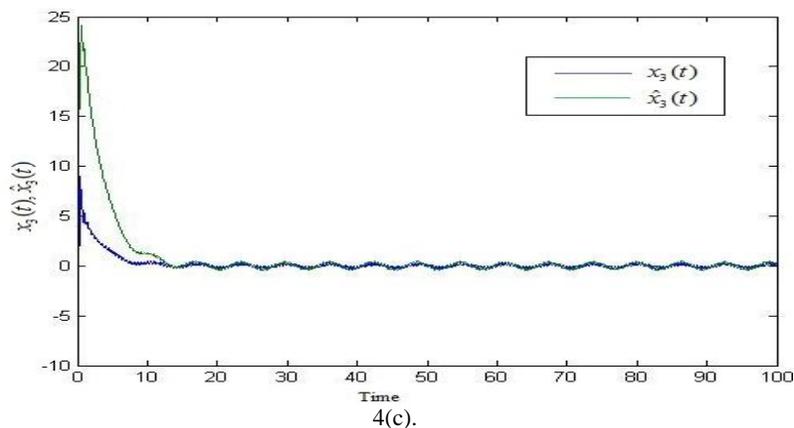


Fig. 4(a)

Fig. 4(b)



4(c).

**Fig. 4 Simulation Results of Estimation of States For Networked UIO**

## V CONCLUSION

NCS has been an active research area over the years due to wide variety of applications. In this paper, a time delay based unknown input observer for networked control system has been proposed.The presence of communication networks adds time delay to the system. For different communication networks, individual respective time delays are considered. The approach of unknown input observer design [1] without time delays (non-networked system) has been reviewed to extend the approach to networked system. The closed loop dynamics for network control system with unknown input observer has been proposed to minimize the effect of time delay and effect of unknown input on state estimation. The simulated results of numerical example shows that the estimated states are almost converging to original states using the proposed approach.

For future work, we would like to extend our results to larger delays plus we would like to take into count the computation time of control law.

## REFERENCES

1. S. Hui, S.H. Zak, "Observer Design for System with Unknown inputs", International Journal of Applied Mathematics and Computer Science, Vol. 15, No.4, pp. 431-446, 2005.

2. Ahmad F. Taha, Ahmed Elmahdi, Jitesh H. Panchal,Dengfeng Sun, "Unknown Input Observer Design and Analysis for Networked Control Systems", International Journal of Control, 2015.

3. D. G. Luenberger," Observers for multivariable systems." IEEE Transactions on Automatic Control, vol. AC-11, no. 2, pp. 190–197, 1966.

4. S. P. Bhattacharyya, "Observer design for linear systems with unknown inputs," IEEE Transactions on Automatic Control, vol. 23, no. 3, pp. 483–484, 1978.

5. M. Darouach, M. Zasadzinski, S. J. Xu, "Full-order observers for linear systems with unknown inputs," IEEE Transactions on Automatic Control, vol. 39, no. 3, pp. 606–609, 1994.

6. N. Kobayashi and T. Nakamizo, "An Observer design for linear systems with unkown inputs," International Journal of Control, vol. 35, pp 605-619, 1982.

7. S. Hui and S. H. ˙Zak, "Observer design for systems with unknown inputs," International Journalof Applied Mathematics and Computer Science, vol. 15, no. 4, pp. 431–446, 2005.

8. Ahmad F. Taha, Ahmed Elmahdi, Jitesh H. Panchal,Dengfeng Sun, "Networked Unknown Input Observer Analysis and Design  for Time Delay Systems", IEEE International Conference on Systems, Man, and Cybernetics,2014.