MULTI-OBJECTIVE STRUCTURAL DESIGN OPTIMIZATION USING GLOBAL CRITERION METHOD

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ABSTRACT

In this paper, we have considered a multi-objective structural optimization problem with several mutually conflicting objectives. We develop an approach for optimizing the design of plane truss structure with multiple objectives subject to a specified set of constraints. In this optimum design formulation, the objective functions are the weight of the truss and the deflection of loaded joint; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. The test problem includes a three-bar planar truss subjected to a single load condition. This multi-objective structural optimization problem is solved by Global criterion method. Numerical example has been given to illustrate our approach.

Keywords: multi-objective non-linear problem, structural design problem, global criterion method

I. INTRODUCTION

Nowadays there are many optimization methods and algorithms have been used in mechanical system and structural design. Structural optimization is a process of minimizing or maximizing an objective function of a structural system. But in the practical structural optimization problem, usually more than one objective are required to be optimized. For example, minimum structural weight, minimum deflection at a specific structural point, maximum stiffness and maximum structural strain energy. Taking note of these, the importance of multi-objective optimization is understood in this field. Several multi-objective approaches for the structural design have been proposed. For example, Dey et al.[1] presented the multi-objective optimal design of three bar truss using fuzzy programming technique. Dey et al.[4] proposed a fuzzy optimization technique for structural optimization using linear and non linear membership function. Dey et at.[2,3] optimized multi-objective structural model using basic T-norm as well as parameterized T-norm based on fuzzy optimization method. Wang et al.[8] used first time α -cut method to optimized non linear structural model. Xu [9] proposed in 1989 two phase method for fuzzy optimization of structures. Shih et at.[7] suggested alternative α -level-cuts methods for structural design optimization problems with fuzzy resources. Huang et al.[5] proposed a fuzzy set based solution method for multi-object optimal design using functional-link net.

Here, we have considered a multi-objective structural optimization problem with several mutually conflicting objectives. We develop an approach for optimizing the design of plane truss structure with multiple objectives subject to a specified set of constraints. In this optimum design formulation, the objective functions are the weight of the truss and the deflection of loaded joint; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. The test problem includes a three-bar planar truss subjected to a single load condition. Numerical example is presented using Global criterion method.

The remainder of this paper is organized in the following way. In section II, we discuss about multi-objective Structural Model. In section III, we discuss Solution of Multi-objective Nonlinear Programming Problem by global criterion method. In section IV, we discuss about Solution of multi-objective structural optimization Problem by global criterion method. In section V, we discuss about numerical solution of structural model of three bar truss. Finally we draw conclusions from the results in section VI.

II. MULTI-OBJECTIVE STRUCTURAL MODEL

In the design of optimal structure i.e. lightest weight of the structure and minimum deflection of loaded joint that satisfies all stress constraints in members of the structure. To bar truss structure system the basic parameters (including the elastic modulus, material density, the maximum allowable stress, etc.) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight, the minimum nodes displacement, in a given load conditions.

The multi-objective Structural model can be expressed as:

$$\begin{aligned} &Minimize\ WT(A) \\ &Minimize\ \delta(A) \\ subject\ to\quad &\sigma(A) \le [\sigma] \\ &A_{\min} \le A \le A_{\max} \end{aligned} \qquad (1)$$

where $A = [A_1, A_2, ..., A_n]^T$ are design variables for the cross section, n is the group number of design variables for the cross section bar, $WT(A) = \sum_{i=1}^{n} \rho_i A_i L_i$ is the total weight of the structure, $\delta(A)$ is the deflection of loaded joint, L_i, A_i and ρ_i were the bar length, cross section area, and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma]$ is maximum allowable stress of the group bars under various conditions, $A_{\min}$ and $A_{\max}$ are the minimum and maximum cross section area respectively.

III. GLOBAL CRITERION METHOD TO SOLVE MULTI-OBJECTIVE NON-LINEAR PROGRAMMING PROBLEM (MONLP)

A Multi-Objective Non-Linear Programming (MONPL) may be taken in the following form:

$$Minimize\ f(x) = [f_1(x), f_2(x), ...f_k(x)]^T$$

(2)

Subject to

$$x \in X = \left\{ x \in R^n : g_j(x) \le or = or \ge b_j \quad for \quad j = 1,2,3,....,m; l_i \le x_i \le u_i, (i = 1,2,3,....,n) \right\}$$

IV. COMPUTATION ALGORITHM FOR GLOBAL CRITERION METHOD TO SOLVE MULTI-OBJECTIVE STRUCTURAL PROBLEM

Step 1. Taking the first objective function from set of objectives of the problem (1) and solve it as a single objective subject to the given constraints. Find the value of objective functions and decision variables.

Step 2.From values of these decision variables compute values of remaining objectives.

Step 3. Repeat the Step 1 and Step 2 for remaining objective functions.

Step 4. After that according to step 3 pay-off matrix formulated as follows:

$$\begin{array}{c} \\ A^1 \\ A^2 \end{array} \begin{array}{c} \begin{array}{cc} WT(A) & \delta(A) \end{array} \\ \begin{bmatrix} WT(A^1) & \delta(A^1) \\ WT(A^2) & \delta(A^2) \end{bmatrix} \end{array}$$

Step 5. The bounds are $U_1 = \max\left\{WT\left(A^1\right), WT\left(A^2\right)\right\}$, $L_1 = \min\left\{WT\left(A^1\right), WT\left(A^2\right)\right\}$ for weight function $WT(A)$ (where $L_1 \le WT(A) \le U_1$) and the bounds of objective are $U_2 = \max\left\{\delta\left(A^1\right), \delta\left(A^2\right)\right\}$, $L_2 = \min\left\{\delta\left(A^1\right), \delta\left(A^2\right)\right\}$ for deflection function $\delta\left(A\right)$ (where $L_2 \le \delta\left(A\right) \le U_2$) are identified.

Using Global criterion method for problem (1), the weighted L_p -problem for minimizing the distances is stated as

$$Minimize\ L_p\left(WT\left(A\right), \delta\left(A\right)\right) = \left(W_1 \left| \frac{U_1 - WT\left(A\right)}{U_1 - L_1} \right|^p + W_2 \left| \frac{U_2 - \delta\left(A\right)}{U_2 - L_2} \right|^p \right)^{1/p}$$

$$subject\ to \quad \sigma(A) \le [\sigma]$$

$$A_{\min} \le A \le A_{\max},\ 1 \le p < \infty.$$

(8) Putting different value of $p\left(1, 2\ or\ \infty\right)$ in (8) we get as follows

For $p = 1$,

$$Minimize\ L_1\left(WT\left(A\right), \delta\left(A\right)\right) = \left(W_1 \left| \frac{U_1 - WT\left(A\right)}{U_1 - L_1} \right| + W_2 \left| \frac{U_2 - \delta\left(A\right)}{U_2 - L_2} \right| \right)$$

$$subject\ to \quad \sigma(A) \le [\sigma]$$

$$A_{\min} \le A \le A_{\max}, W_1 + W_2 = 1.$$

(9) .For $p = 2$

$$Minimize\ L_1\left(WT(A),\delta(A)\right)=\left(W_1\left|\frac{U_1-WT(A)}{U_1-L_1}\right|^2+W_2\left|\frac{U_2-\delta(A)}{U_2-L_2}\right|^2\right)^{1/2}$$

$$subject\ to\quad \sigma(A)\le[\sigma]$$

$$A_{\min}\le A\le A_{\max},W_1+W_2=1.$$

(10)

For $p=\infty$,(8) is of the form

$$Minimize\ \lambda$$

$$subject\ to\ W_1\left|\frac{U_1-WT(A)}{U_1-L_1}\right|\le\lambda$$

$$W_2\left|\frac{U_2-\delta(A)}{U_2-L_2}\right|\le\lambda$$

$$\sigma(A)\le[\sigma]$$

$$A_{\min}\le A\le A_{\max},\lambda\in\Re,W_1+W_2=1.$$

(11)

To solve the structural optimization problem (1) using GCM, we have to solve (9),(10),(11) with same constraints as in equation (1) for different weight.

V. NUMERICAL ILLUSTRATION

A well-known three bar [7] planar truss structure is considered. The design objective is to minimize weight of the structural $WT(A_1,A_2)$ and minimize the vertical deflection $\delta(A_1,A_2)$ at loading point of a statistically loaded three-bar planar truss subjected to stress $\sigma_i(A_1,A_2)$ constraints on each of the truss members $i=1,2,3$.

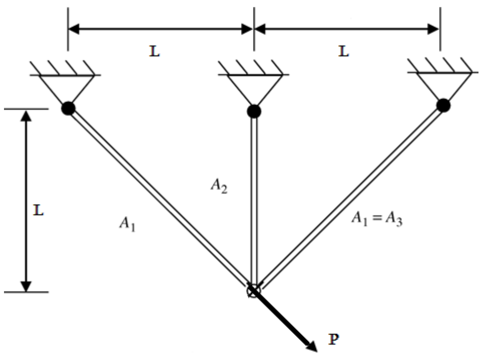

Figure 1: Design of the three-bar planar truss

The multi-objective optimization problem can be stated as follows:

$$\begin{aligned}
&\text{minimze } WT(A_1,A_2)=\rho L\left(2\sqrt{2}A_1+A_2\right),\\
&\text{minimize } \delta(A_1,A_2)=\frac{PL}{E\left(A_1+\sqrt{2}A_2\right)},\\
&\text{Subject to } \sigma_1(A_1,A_2)\equiv\frac{P\left(\sqrt{2}A_1+A_2\right)}{2A_1A_2+\sqrt{2}A_1^2}\le[\sigma_T],\\
&\sigma_2(A_1,A_2)\equiv\frac{P}{A_1+\sqrt{2}A_2}\le[\sigma_T],\\
&\sigma_3(A_1,A_2)\equiv\frac{PA_2}{2A_1A_2+\sqrt{2}A_1^2}\le[\sigma_C],\\
&A_i^{\min}\le A_i\le A_i^{\max}, i=1,2.
\end{aligned}$$

(12)

The input data for MOSOP (12) is given as follows:

Applied load P (KN)	Material density ρ (KN/m^3)	Length L (m)	Maximum allowable tensile stress σ_T (KN/m^2)	Maximum allowable compressive stress σ_C (KN/m^2)	Young's modulus E (KN/m^2)	$A_i^{\min}$ and $A_i^{\max}$ of cross section of bars $(10^{-4}m^2)$
20	100	1	20	15	2×10^8	$A_i^{\min}=0.1$ $A_i^{\max}=5; i=1,2$

Solution: According to step 2 pay off matrix is formulated as follows:

$$\begin{array}{c} \\ A^1 \\ A^2 \end{array}\begin{array}{cc} WT(A_1,A_2) & \delta(A_1,A_2) \\ \end{array}$$

$$\begin{array}{c} A^1 \\ A^2 \end{array}\begin{bmatrix} 2.638958 & 14.64102 \\ 19.14214 & 1.656854 \end{bmatrix}$$

Here $U_1=19.14214$, $L_1=2.638958$, $U_2=14.64102$, $L_2=1.656854$ are indentified.

The optimal solutions of the multi-objective structural optimization model (12) using global criterion method (following (9),(10) and (11)) are given in table 1, table 2 and table 3 for different preference values of the objective functions.

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Case I: Table 1 shows different optimal solutions when the decision maker supplies more preference to the deflection function than the weight function. Here deflection $\delta(A_1, A_2)$ is minimum when $p \to \infty$ whereas weight $WT(A_1, A_2)$ is minimum when $p = 1$

Case II: Table 2 shows different optimal solutions then decision maker supplies equal preferences to the weight function and deflection function. Here deflection $\delta(A_1, A_2)$ is minimum when $p = 1$ whereas weight $WT(A_1, A_2)$ is minimum when $p \to \infty$

Case III: Table 3 shows different optimal solutions when the decision maker supplies more preference to the weight function than the deflection function. Here deflection $\delta(A_1, A_2)$ is minimum when $p = 1$ whereas weight $WT(A_1, A_2)$ is minimum when $p \to \infty$.

Table 1: Optimal solution for different weightages of weight of structure and deflection of the nodal point, $W_1 = 0.2, W_2 = 0.8$

p	$A_1 \times 10^{-4} m^2$	$A_2 \times 10^{-4} m^2$	$WT(A_1, A_2) \times 10^2 KN$	$\delta(A_1, A_2) \times 10^{-7} m$
1	1.224604	0.1	3.56702	14.64102
2	1.292400	0.1	3.755459	13.94874
∞	1.558851	0.1	4.509098	11.76282

Table 2: Optimal solution for different weightages of weight of structure and deflection of the nodal point, $W_1 = 0.5, W_2 = 0.5$

p	$A_1 \times 10^{-4} m^2$	$A_2 \times 10^{-4} m^2$	$WT(A_1, A_2) \times 10^2 KN$	$\delta(A_1, A_2) \times 10^{-7} m$
1	5	5	19.14214	1.656854
2	5	0.1	14.24124	3.889975
∞	3.007374	0.1	8.606137	6.351636

Table 3: Optimal solution for different weightages of weight of structure and deflection of the nodal point, $W_1 = 0.8, W_2 = 0.2$

p	$A_1 \times 10^{-4} m^2$	$A_2 \times 10^{-4} m^2$	$WT(A_1, A_2) \times 10^2 KN$	$\delta(A_1, A_2) \times 10^{-7} m$
1	5	5	19.14214	1.656854
2	5	3.568181	17.17032	1.990809
∞	5	1.280763	15.42290	2.936309

VI. CONCLUSIONS

Here structural model with weight and node displacement objectives are presented. The multi-objective structural problem is solved by Global criterion method. Two objective functions are combined into a single objective function by the Global criterion method. The optimal solutions for different preferences on objective functions are presented. Decision-maker may obtain the Pareto optimal results according to his/her expectation of structural design. A main advantage of the proposed method is that it allows the user to concentrate on the actual limitations in a problem during the specification of the flexible objectives. This approximation method can be applied to optimize different models in various fields of engineering and sciences.

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