

FORECASTING FUTURE DATA OF WHEAT AND PADDY PRODUCTION OF PUNJAB BASED ON ERROR ESTIMATION

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ABSTRACT

This study evaluated measures for making comparisons of errors using time series. Many measures of forecast accuracy have been proposed in the past, and several authors have made recommendations about what should be used when comparing the accuracy of forecast methods applied to time series data. . In this paper statistical time series modelling techniques like moving average and least square method are used to study the future requirement of paddy and wheat and their performances are evaluated in terms of Mean absolute error (MAE), Mean square error (MSE) and Mean absolute percentage error(MAPE).

Keywords- Least square method, Mean absolute error (MAE), Mean Square error (MSE) , Mean absolute percentage error (MAPE)and Moving average method

I. INTRODUCTION

Forecasts of agricultural production and prices are intended to be useful for farmers, governments, and agribusiness industries. Because of the special position of food production in a nation's security, governments have become both principal suppliers and main users of agricultural forecasts. They need internal forecasts to execute policies that provide technical and marketing support for the agricultural sector.

Generally forecasting techniques used past or historical data in the form of time series. A time series can be defined as an ordered sequence of random variables over time . It is the historical record of any activity , with observation taken at equally spaced intervals. Many measures on forecast accuracy have been proposed in the past, and several authors have made recommendations about what should be used when comparing the accuracy of forecast methods applied to univariate time series data. It is our contention that many of these proposed measures of forecast accuracy are not generally applicable in reality are found infinite or undefined, and can produce misleading results. The authors like Brockwell and Davis [2],[4] and Michael Lawrence[9] et al. Recently provides a new series of work on time series forecasting. In this paper, we provide our own recommendations of forecasting future data of paddy and wheat production of Punjab, India and provide the future demand of this product and also provide an empirical comparison.

The agricultural sector is facing important global challenges due to the pressure of food demand, increased price competition produced by market globalization and food price volatility , and the necessity of more

environmentally and economically sustainable farming. Recently Becker Reshef et al.[15] and later S. Skakun et. al.[3],[15] provided a detail study of wheat production and its forecasting values[11],[16].

Over the past-two decades, many studies have been conducted to identify the appropriate method for finding the most accurate forecasts for a given class of time series. Such generalizations are important because organizations often rely upon a single method for data. Thus, it is particularly a problem when trying to specify the best method for a well-defined set of conditions, Thus it is important to identify which error measures are useful under given series. Thus error measures play an important role in refining a time series model such that it will be an accurate forecasting error for a given set of data.

Various time series models were developed by many authors for average yield for the concerned study area. Moving average method is the most used historical method for forecasting future data. Moving averages are developed at an average of weighted observations, which tends to smooth out short-term irregularity in the data series. Moving Average method has the great merit of flexibility involving construction by taking averages of several sequential values of another time series[10]. It is a type of mathematical convolution. The greater number of periods in the moving average the greater the smoothing effect. In this regard the authors like Robert, J Hatchett [13] and Rob, J. Hyndman [14],[19] has done an influential work recently.

The least square method is a mathematical optimization technique, which is used to infer the best function matching relationship between the discrete data by the square of the difference between the measured value and the actual value[6],[7] . In engineering technology and scientific experiment, the data set is often obtained. Determining the appropriate function curve according to the data is the main problem of curve fitting in experimental data processing. Curve fitting method is the most commonly used methods and the fitting of the piecewise curve is better than the general curve fitting effect . In this regard a lot of work has already been done. So far, a lot of practical curve fitting methods have been put forward.

Error measurement statistics play a critical role in tracking forecast accuracy, monitoring for exceptions, and benchmarking your forecasting process[8],[12]. Interpretation of these statistics can be tricky, particularly when working with low-volume data or when trying to assess accuracy across multiple items[1] . Many measures of forecast accuracy have been proposed in the past, and several authors have made recommendations about what should be used when comparing the accuracy of forecast methods applied to time series data[5]. Although various stochastic and deterministic measures are available in the literature to evaluate the performance of forecasting techniques, in this paper the MSE, MAE and MAPE are used for our convenience[18],[20]. The main emphasis of this work is to compare the various forecasting techniques[8]. In this manuscript an attempt also has been made to forecast the production by using the Moving Average Method and Least Square Method. The aim is to evaluate the performances in terms of MAE,MSE and MAPE. Finally, we compare the findings and decide the suitability among the methods MAE, MSE and MAPE and also with the classic methods like MA and Least square method.

In this paper, a procedure for applying time series analysis to forecast production is described. The moving average and least square method are applied to our given set of data based on the production of wheat and paddy of Punjab. Our study is mainly based on the deterministic measures like Mean absolute error (MAE), Mean square error (MSE) and Mean absolute percentage error(MAPE).

Section 2 provides the detail of the source of our data , In section 4.1, the result of moving average is given, section 4.2 provides detail of evaluating the forecast accuracy based on moving average whereas in section 4.3 the obtained results of the least square method are provided elaborately. Next, in section 5 and 6 detailed discussion of results are discussed and finally a conclusion is given regarding forecasting procedure.

II. METHODOLOGY

A moving average is a technique to get an overall idea of the trends in a data set; it is an average of any subset of numbers. The moving average is extremely useful for forecasting long-term trends. We can calculate it for any period of time. The formula for moving average is

$$F_t = \sum_{n=1}^n \frac{A_{t-n}}{n}$$

Where F_t = Forecast for the coming period , n = Number of period to be averaged

In the past period, two or three periods ago and so on respectively .Equal weighting is given to each of the values used in the moving average calculation. Whereas it is reasonable to suppose that the most recent data is more relevant to current conditions .The moving average calculation takes no account of data outside the period of average, so full use is not made of all the data available.

Least squares method is one of the statistical methods used to find out the line of best fit for a model where the line of best fit is such that it minimizes the sum of squares of the distances of the points from this line. The points represent observed data values whereas the best fit line will give a statistical model for the process. The formula for least square method is

$$Y_c = a + bx$$

Where Y_c = commuted value of the trend , x = independent variable which represents time , a =Value of trend when x is zero (y intercept) , b = slope of the line. Here 'a' and 'b' are constants .Once their values are determined they do not change. The value of 'b' represents the amount by which the trend increases or decreases for each unit of time. The values of 'a' and 'b' are computed by solving a set of simultaneous equations, commonly called normal equations Normal equations are:

$$\begin{aligned}\sum y &= Na + b \sum x \\ \sum xy &= a \sum x + b \sum x^2\end{aligned}$$

In above equations, y represents original values in a time series, N stands for the number of years and x represents time.

There are many ways to measure forecast accuracy. Some of these measures are the mean absolute error (MAE), mean square error (MSE) and the mean absolute percentage error (MAPE). These error estimates helps in monitoring erratic demand observations. In addition they also help to determine when the forecasting method or no longer tracking actual demand and it need to be reset. The mathematical formulas

Error= Actual value- Forecast value

Mean absolute error = $|Error|$

Mean Square error = $|Error|^2$

$$\text{Mean absolute percentage error} = \frac{\text{Error}}{\text{Actual value}} \times 100$$

III. DETAILS OF DATA

In this study we used the data of wheat and paddy production for the period 2005-06 to 2016-17. Data obtained from Punjab Mandi Board, Mohali, Chandigarh. Moving average and Least square methods are applied and their performance evaluated in terms MAE, MSE and MAPE.

Table(1): Recorded wheat and paddy production data of Punjab Mandi Board , Mohali , Chandigarh

S.NO.	YEAR	PADDY(In lakh tones)	WHEAT(In lakh tones)
1	2005-06	25.18	10.53
2	2006-07	28.61	10.52
3	2007-08	26.42	12.6
4	2008-09	36.01	18.4
5	2009-10	41.31	16.69
6	2010-11	40.7	16.95
7	2011-12	33.54	17.51
8	2012-13	37.92	22.08
9	2013-14	37.12	20.24
10	2014-15	36.8	22.07
11	2015-16	35.6	15.9
12	2016-17	36.5	18.95

IV. RESULT ANALYSIS

In this section, we used the data for wheat and paddy production for the period 2005 to 2016-17. The various methods are applied one by one and their performances are evaluated in terms of MAE, MAPE and MSE.

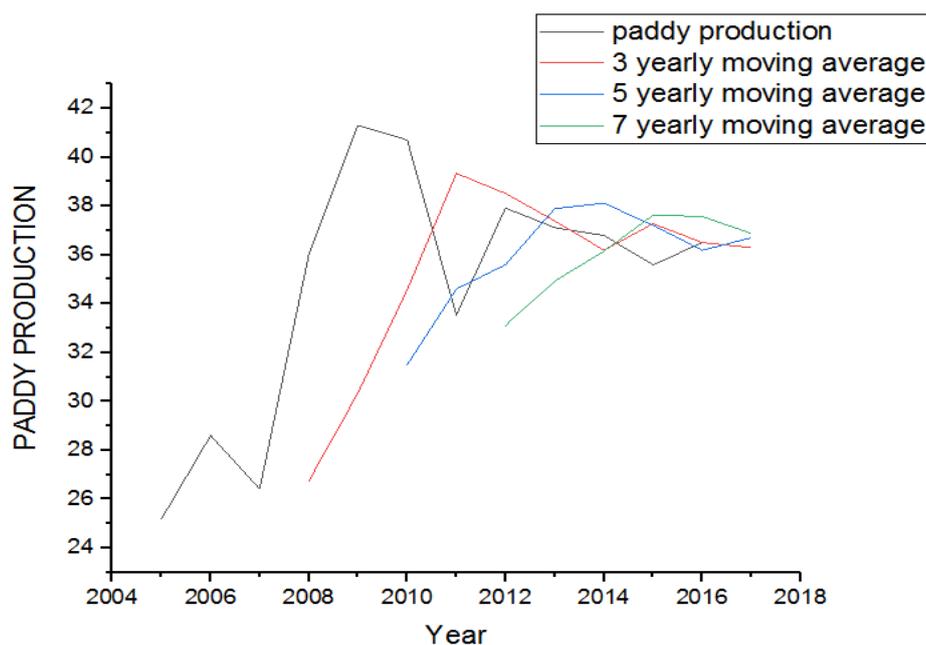
4.1 Moving Average Method

In this section at first we obtain the Moving average results by taking 3 types of data span. Table 2 shows actual and estimated values of paddy production by 3 yearly , 5 yearly and 7 yearly moving average. The figure 1 shows the same results graphically for better understanding of the smoothness of the data. Similarly Table 3 and figure 2 shows the result of moving average by taking the same length of year for wheat production and also provide its smoothness based on moving averages.

Table (2): Actual and estimated values of paddy production by Moving average

Sr. No.	Year	Paddy(In lakh tones)	3 yearly moving average	5yearly moving average	7 yearly moving average
1	2005-06	25.18			
2	2006-07	28.61			
3	2007-08	26.42			
4	2008-09	36.01	26.74		
5	2009-10	41.31	30.35		
6	2010-11	40.70	34.58	31.51	
7	2011-12	33.54	39.34	34.61	
8	2012-13	37.92	38.52	35.60	33.11
9	2013-14	37.12	37.39	37.90	34.93
10	2014-15	36.80	36.19	38.12	36.15
11	2015-16	35.60	37.28	37.22	37.63
12	2016-17	36.5	36.51	36.20	37.57
13	2017-18		36.30	36.79	36.88

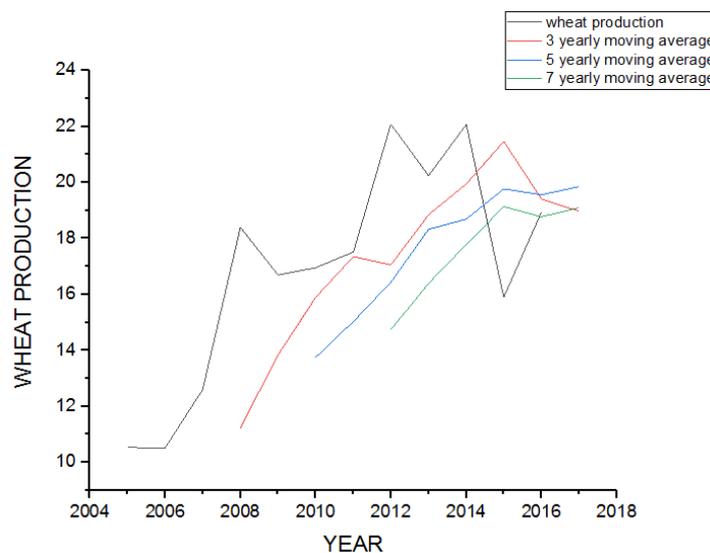
Figure(1):The trend of moving average of paddy production



Table(3) :Actual and estimated values of wheat production by Moving average

Sr. No.	Year	Wheat (In lakh tones)	3 yearly moving average	5 yearly moving average	7 yearly moving average
1	2005-06	10.53			
2	2006-07	10.52			
3	2007-08	12.6			
4	2008-09	18.4	11.22		
5	2009-10	16.69	13.84		
6	2010-11	16.95	15.90	13.75	
7	2011-12	17.51	17.35	15.03	
8	2012-13	22.08	17.05	16.43	14.74
9	2013-14	20.24	18.85	18.33	16.39
10	2014-15	22.07	19.94	18.69	17.78
11	2015-16	15.9	21.46	19.77	19.13
12	2016-17	18.95	19.4	19.56	18.78
13	2017-18		18.97	19.8	19.1

Figure(2):The trend of moving average of wheat production



4.2 Evaluating the Forecast Accuracy based on Moving Average

There are many ways to measure forecast accuracy. Some of these measures are the mean absolute forecast error (MAE), the mean absolute percentage error (MAPE) and the mean square error (MSE). This error estimate helps in monitoring erratic demand observations. In addition, they also help to determine the forecasting method. The mean absolute error (MAE) is important because of its simplicity and usefulness in obtaining adequate result. MAE is the average error in the forecasts, based on absolute values. It is like the standard deviation, measures the dispersion of some observed value from some expected value. The only difference is that like standard deviation, the errors are not squared. Standard errors square root of a function, it is often more

convenient to use the function itself. This is called the mean square error (MSE) or variance. The mathematical formulas may be used while evaluating data are

Error= Actual value- Forecast value

Mean absolute error = $|Error|$

Mean Square error = $|Error|^2$

Mean absolute percentage error = $\frac{Error}{Actual\ value} \times 100$

Table(4): Moving Average based on 3, 5 and 7 yearly of paddy production

Sr	Year	Paddy(In lakh tones)	3 yearly moving average				5 yearly moving average				7 yearly moving average							
			Forecast	error	absolute error	squared error	absolute percentage error	Forecast	error	absolute error	squared error	absolute percentage error	Forecast	error	absolute error	squared error	absolute percentage error	
1	2005-06	25.18																
2	2006-07	28.61																
3	2007-08	26.42																
4	2008-09	36.01	26.74	9.27	9.27	85.933	25.75											
5	2009-10	41.31	30.35	11	10.96	120.12	26.53											
6	2010-11	40.7	34.58	6.12	6.12	37.454	15.03	31.51	9.19	17.46	304.85	42.89						
7	2011-12	33.54	39.34	-5.8	5.8	33.64	17.29	34.61	-1.07	1.07	1.14	3.19						
8	2012-13	37.92	38.52	-0.6	0.6	0.36	1.58	35.6	2.32	2.33	5.42	6.14	33.11	4.81	4.88	23.81	12.86	
9	2013-14	37.12	37.39	-0.3	0.27	0.0729	0.73	37.9	-0.78	0.77	0.59	2.07	34.93	2.19	2.26	5.1	6.08	
10	2014-15	36.8	36.19	0.61	0.61	0.3721	1.66	38.12	-1.32	1.31	1.71	3.55	36.15	0.65	0.66	0.43	1.79	
11	2015-16	35.6	37.28	-1.7	1.68	2.8224	4.7191011	37.22	-1.62	1.62	2.6244	4.5505618	37.63	-2.03	2.03	4.1209	5.7022472	
12	2016-17	36.5	36.51	-0.01	0.01	0.0001	0.0273973	36.2	0.3	0.3	0.09	0.8219178	37.57	-1.07	1.07	1.1449	2.9315068	
13	2017-18		36.30					36.79					36.88					
Total					35.32	280.78	93.316498			24.86	316.424	63.21248			10.9	34.6058	29.363754	
AN OF ERRORS					3.92444	31.197	10.3685			3.55143	45.2035	9.0303542			2.18	6.92116	5.8727508	

Table(5): Moving Average based on 3, 5 and 7 yearly of wheat production

Sr	Year	Wheat(In lakh tones)	3 yearly moving average				5 yearly moving average				7 yearly moving average							
			Forecast	error	absolute error	squared error	absolute percentage error	Forecast	error	absolute error	squared error	absolute percentage error	Forecast	error	absolute error	squared error	absolute percentage error	
1	2005-06	10.53																
2	2006-07	10.52																
3	2007-08	12.6																
4	2008-09	18.4	11.22	7.18	7.18	51.552	39.021739											
5	2009-10	16.69	13.84	2.85	2.85	8.1225	17.076093											
6	2010-11	16.95	15.9	1.05	1.05	1.1025	6.1946903	13.75	3.2	3.2	10.24	18.879056						
7	2011-12	17.51	17.35	0.16	0.16	0.0256	0.9137636	15.03	2.48	2.47	6.1009	14.106225						
8	2012-13	22.08	17.05	5.03	5.03	25.301	22.780797	16.43	5.65	5.65	31.9225	25.588768	14.74	7.34	7.33	53.7289	33.197464	
9	2013-14	20.24	18.85	1.39	1.39	1.9321	6.8675889	18.33	1.91	1.91	3.6481	9.4367589	16.39	3.85	3.84	14.7456	18.972332	
10	2014-15	22.07	19.94	2.13	2.13	4.5369	9.6511101	18.69	3.38	3.37	11.3569	15.269597	17.78	4.29	4.28	18.3184	19.392841	
11	2015-16	15.9	21.46	-5.6	5.6	31.36	35.220126	19.77	-3.87	3.87	14.9769	24.339623	19.13	-3.23	3.23	10.4329	20.314465	
12	2016-17	18.95	19.4	-0.4	0.4	0.16	2.1108179	19.56	-0.61	0.61	0.3721	3.2189974	18.78	0.17	0.17	0.0289	0.8970976	
13	2017-18							19.85					19.1					
Total					25.79	124.09	139.83673			21.08	78.6174	110.83902			18.85	97.2547	92.7742	
AN OF ERRORS					2.86556	13.788	15.537414			3.32	12.6537	15.834146			3.77	19.4509	18.55484	

The above two tables (e.g. Tab 4 , Tab 5) shows the evaluated errors measures of paddy and wheat production.

Table(6):Error Measure of Moving Average Method

Error Measures	3 yearly Moving Average		5 yearly Moving Average		7 yearly Moving Average	
	Paddy	Wheat	Paddy	Wheat	Paddy	Wheat
MAE	3.92	2.87	3.55	3.32	2.18	3.77
MSE	31.2	13.79	45.2	12.65	6.92	19.45
MAPE	10.3%	15.5%	9.03%	15.83%	5.87%	18.5%

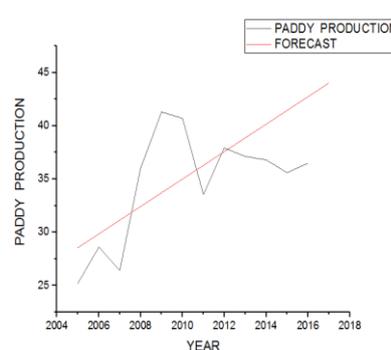
Table 6 shows the error measures of MA methods by applying the error measures MAE ,MSE & MAPE. After analysis of this table value we shows that 3 yearly moving average has minimum errors in production of paddy.7 yearly moving average has minimum errors for wheat production.

4.3 Least Square Method and error measures based on this method: In this section at first we obtain the Least Square Method results by using formulas. Table(7) shows actual and estimated values of paddy production by least square method. The figure(3) shows the same results graphically for better understanding of the smoothness of the data. Similarly Table (8) and figure (4) shows the result of least square method by taking the same length of year for wheat production and also provide its smoothness based on least square method.

Table(7) Actual and estimated values of paddy production by least square method

X	Paddy(Y)	XY	X ²	Forecast	Error	Absolute Error	Squared Error	Absolute Percentage Error
0	25.18	0	0	29.6	-4.42	4.42	19.5364	13.38
1	28.61	28.61	1	30.55	-1.94	1.94	3.7636	4.29
2	26.42	52.84	4	31.5	-5.08	5.08	25.8064	17.82
3	36.01	108.03	9	32.45	3.56	3.56	12.6736	9.96
4	41.31	165.24	16	33.4	7.91	7.91	62.5681	18.39
5	40.7	203.5	25	34.35	6.35	6.35	40.3225	14
6	33.54	201.24	36	35.3	-1.76	1.76	3.0976	8.19
7	37.92	265.44	49	36.25	1.67	1.67	2.7889	0.89
8	37.12	296.96	64	37.2	-0.08	0.08	0.0064	4.71
9	36.8	331.2	81	38.15	-1.35	1.35	1.8225	9.13
10	35.6	356	100	39.1	-3.5	3.5	12.25	9.83
11	36.5	401.5	121	40.05	-3.55	3.55	12.6025	9.73
12	42.74	512.88	144	41	1.74	1.74	3.0276	4.07
total n=78	458.45	2923.44	650	458.9		42.91	200.2661	124.39
	a=29.6	b=0.95						
MEAN OF ERRORS						3.300769	15.405085	9.57
		F=a+bx						
		F=29.6+0.95x						

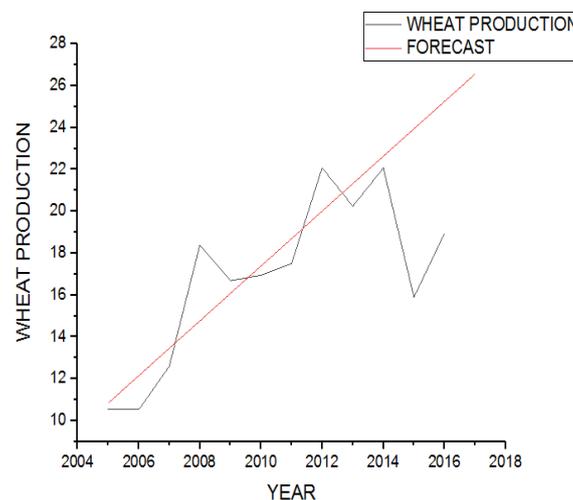
Figure(3): The trend of least square method of paddy production



Table(8) Actual and estimated values of wheat production by least square method

X	Wheat (Y)	XY	X ²	Forecast	Error	Absolute error	Squared Error	Absolute Percentage Error
0	10.53	0	0	12	-1.47	1.47	2.16	13.96
1	10.52	10.52	1	12.92	-2.4	2.4	5.76	22.81
2	12.6	25.2	4	13.84	-1.24	1.24	1.54	9.84
3	18.4	55.2	9	14.76	3.64	3.64	13.25	19.78
4	16.69	66.76	16	15.68	1.01	1.01	1.02	6.05
5	16.95	84.75	25	16.6	0.35	0.35	0.12	2.06
6	17.51	105.06	36	17.52	-0.01	0.01	0.00	0.06
7	22.08	154.56	49	18.44	3.64	3.64	13.25	16.49
8	20.24	161.92	64	19.36	0.88	0.88	0.77	4.35
9	22.07	198.63	81	20.28	1.79	1.79	3.20	8.11
10	15.9	159	100	21.2	-5.3	5.3	28.09	33.33
11	18.95	208.45	121	22.12	-3.17	3.17	10.05	16.73
12	25.25	303	144	23.04	2.21	2.21	4.88	8.75
total=78	227.69	1533.05	650				84.10	162.33
	a=12	b=0.92						
MEAN OF ERRORS						2.09	6.47	12.49
		F=a+bx						
		F=12+0.92x						

Figure(4):The trend of least square method of wheat production



By comparing the tables 7 and 8 the performance of the methods by Moving Average and Least square method on the basis of MAE ,MSE and MAPE the following table is found which has less error and more accuracy than the other method.

Table(9):Error Measure of Least Square Method

Crops	Paddy	Wheat
Error Measures		
MAE	3.3	2.09
MSE	15.41	6.47
MAPE	9.57%	12.49%

After evaluating the error measures, next we are going to discuss our results in the following section .

V. DISCUSSION

In the previous section, two valid methodologies namely Moving average and Least square methods are used to predict future data. In section 3 detail of wheat and paddy production of Punjab Mandi board , Mohali Chandigarh for the period of 2005-06 to 2016-17 is provided for our study concern. In this paper we forecast the future data of both, paddy and wheat for the period of 2017-18. For this purpose we applied the methodology of minimum error method. In this study we ignore the large errors, which are the sometime primary concern. But sometimes large errors created disproportionate impacts for forecasting future data. So the selection of an error measure is dependent upon the situation. None of the error measures was superior on all criteria. In this paper the performance of the various methods are evaluated on the basis of MAE, MSE and MAPE .

Table 10: Diagnostic measures for the selection of the best forecasting method for wheat and paddy production

Crops	Paddy		Wheat	
Method	7MA	LS	3MA	LS
MAE	2.18	3.3	2.87	2.09
MSE	6.92	15.4	13.79	6.47
MAPE	5.87%	9.57%	15.5%	12.49%

The above table shows errors of paddy and wheat production on the basis of MAE, MSE and MAPE. In case of production of paddy the error value of moving average on the basis of MAE , MSE and MAPE are less as compare to least square method. On the other hand for the production of wheat, the error values of least square method are smaller than the moving average method on the basis of MAE, MSE and MAPE .

By comparing the performance of these method it has been found that according to data set we have selected the best method for forecast the future data. Therefore in this study we select the moving average method is appropriate for forecasting the future data of paddy production and on the other hand the least square method is appropriate for forecasting the wheat production and it is because of their least error measures in terms of MAE, MSE and MAPE are less.

VI. CONCLUSION

In this paper, an attempt is made to obtain forecast of paddy and wheat production , Punjab, India. Analysis is based on time series of past production. We explored the impact of using methodologies based on error measures. Forecasting of wheat and paddy production done by using statistical methods (Moving average Method, Least square method). Statistical method are chosen because of their rich historic data and ease of their use . Finally their performance evaluated by comparing the MAE, MSE and MAPE obtained from the different methods .The result shows that the moving average method is more accurate average method for the production of paddy whereas the production of wheat we find least square method is more accurate method on the basis of error estimation process .

The forecasting technique may be differed for different areas . It depends on variable factor. Hence this work may be extended to other agriculture areas.

VII. ACKNOWLEDGEMENT

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