FLOW OF ROTATING FLUID PAST AN ACCELERATED SURFACE EMBEDDED IN POROUS MEDIUM WITH RADIATION AND CHEMICAL REACTION

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ABSTRACT

Simultaneous heat and mass transfer in unsteady free convection flow of rotating fluid with thermal radiation and first order chemical reaction, is present when the temperature at the plate oscillates with time. A uniform magnetic field is assumed to be applied along the normal to the plate. The dimensionless governing equations for this investigation are solved analytically using Laplace transform technique (in closed form). A parametric study illustrating the influence of various parameters on velocity, temperature as well as on concentration is conducted. The results of this parametric study are shown graphically.

keywords: Chemical Reaction, MHD, Radiation, Rotating fluid.

I. INTRODUCTION

As the problem of flow, heat and mass transfer of an incompressible viscous fluid in a porous medium has an important bearing on several applications in the field of metallurgy and chemical engineering, so, in recent years, considerable attention has been received on the study of boundary layer flow behavior and heat mass transfer characteristic of Newtonian fluid past a vertical plate embedded in porous medium in the presence of magnetic field because of its wide spectrum of applications in engineering processes, especially in the enhanced recovery of petroleum resources, plasma studies, drying of porous solid, thermal insulation and MHD generators. In addition to the above, the phenomenon of heat mass transfer is also encountered in food processing and polymer production. There has been renewed interest in studying MHD flow and heat transfer in porous media due to the effect of magnetic fields on flow control and on the performance of many systems using electrically conducting fluids. Unsteady natural convection heat and mass transfer flow along infinite vertical plate have been studied by several authors, imposing different physical conditions. Jha and Prasad [1] studied heat source characteristics on free convection and mass transfer flow past an impulsively started infinite vertical plate through porous medium under the action of transversely applied magnetic field. Helmy [2] analyzed the unsteady free convection flow past a vertical plate. Chamkha [3] investigated unsteady heat and mass transfer past an infinite vertical permeable moving plate in porous medium with heat absorption. The plate velocity was maintained at a constant value and the flow was subjected to a transverse magnetic field. Combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium has been studied by Chaudhary and Jain [4]. Recently Chaudhary and Jain [5] studied magnetohydrodynamic convection flow past an accelerated surface embedded in porous medium; they solved the problem analytically using Laplace transform technique. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in geophysical fluid dynamics. It is also important in the solar physics dealing with the sunspot development, the solar cycle and the structure of rotating magnetic stars. Many authors have studied hydro magnetic rotating fluid flows under different physical conditions some of them are Debnath [6], Bestman and Adjepong [7], Takhar et.al.[8], Mbeledogu and Ogulu [9], Singh et.al [10] and Ahmed and Kalita [11]. Deka et.al. [12] investigated the effect of first order chemical reaction on flow past impulsively started infinite vertical plate with constant heat flux. Muthukumarswamy and Ganesan [13] discussed effects of chemical reaction on flow characteristic in an unsteady upward motion of an isothermal plate. MHD flow of uniformly stretched vertical surface in the presence of heat absorption/generation and a chemical reaction has been considered by [14]. Many processes in engineering occurs at high temperature and knowledge of radiation heat transfer becomes very important for design of reliable equipments, gas turbines, nuclear plants and various propulsion devices, based on these applications Cogley [15] showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the boundaries. Ogulu and Cookey [16] and Cookey et.al. [17] studied influence of radiation on MHD free convection flow past an infinite vertical plate in a porous medium with time dependent suction. Free convection flow with thermal radiation and mass transfer past a moving vertical plate has been analyzed by Makinde [18]. Ibrahim et.al. [19] investigated the effect of the chemical reaction and radiation absorption on the MHD free convection flow past a semi–infinite vertical moving plate.

It is proposed here to study the heat and mass transfer flow by MHD natural convection past a vertical moving plate when the temperature of the plate oscillates in time about a constant mean temperature and the plate is embedded in porous medium.

This paper is the extension of the work of Chaudhary and Jain [5] in which we have introduced mass transfer for a rotating fluid with first order chemical reaction and radiative heat transfer.

The present work may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas filed, and water purification processes.

II. FORMULATION OF THE PROBLEM

We consider an unsteady flow of an incompressible viscous electrically conducting fluid which moves in its on plane with velocity V_0 and rotates with angular velocity Ω. The z- axis is taken normal to the plate. Due to infinite assumptions, the flow variations are function of z and t only, besides the analysis is based on the following assumption.

(i) The uniform magnetic field of small magnetic Reynolds number acts transversely to the direction of the flow.
(ii) Hall Effect, the polarization and viscous dissipation effect are ignored.
(iii) The induced magnetic field is negligible.

(iv) The radiative heat flux term is simplified by using Rosselands approximation.

(v) It is also assumed that the chemically reactive species undergo first order irreversible chemical reaction.

Under these assumptions the usual Boussinesq approximations, the proposed governing equations are.

Linear momentum equation

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = v\frac{\partial^2 u'}{\partial z'^2} + g\beta_T(T - T_\infty) + g\beta_c(C - C_\infty) - \frac{v}{k'}u' - \frac{\sigma}{\rho}B_0^2 u' \quad \dots (1)$$

Angular momentum equation

$$\frac{\partial v'}{\partial t'} - 2\Omega' u' = v\frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma\mu^2 H_0^2 v'}{\rho_\infty} \quad \dots (2)$$

Energy equation

$$\rho_\infty c_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial z'^2} - \frac{\partial q'_r}{\partial z'} \quad \dots (3)$$

$$\rho_\infty c_p \frac{\partial c'}{\partial t'} = D_m \frac{\partial^2 c'}{\partial z'^2} - k_r' c \quad \dots (4)$$

Corresponding boundary conditions are

$$u' = U_0, v' = 0, T = 1 + \varepsilon\cos\omega t, c' = c_w \ \ on\, z' = 0$$
$$u' = 0, v' = 0,, T = 0, c' = c_\infty \ \ on\, z' \to \infty \quad \dots (5)$$

Now introducing the following non dimensional quantities and parameters

$$z' = \frac{vz}{U_0}, y = \frac{Y}{d}, u' = U_0 u, v' = vU_0, \theta = \frac{T - T_w}{T - T_\infty}, C = \frac{c - c_w}{c - c_\infty}$$
$$K' = \frac{v^2 K}{U_0^2}, \Omega' = \frac{U_0^2\Omega}{v}, q_r' = \frac{U_0 k q_r}{\rho c_p}, Sc = \frac{\mu c_p}{D_m}$$
$$\Pr = \frac{\mu c_p}{k}, M^2 = \frac{\sigma\mu v H_0^2}{\rho_\infty U_0^2}, Gr = \frac{g\beta v(T - T_\infty)}{U_0^3}, Gc = \frac{g\beta v(c - c_\infty)}{U_0^3} \quad \dots (6)$$

Using Eq. (6) into Eq.(1)-(5) we obtain

$$\frac{\partial u}{\partial t} - 2\Omega v = \frac{\partial^2 u}{\partial z^2} + Gr\theta + G_C C - \frac{u}{k'} - M^2 u \quad \dots (7)$$

$$\frac{\partial v}{\partial t} - 2\Omega u = \frac{\partial^2 v}{\partial z^2} - M^2 v \quad \dots (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial z^2} - \frac{\partial q_r}{\partial z} \quad \dots (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2} - k_r C \quad \ldots (10)$$

III. SOLUTION OF THE PROBLEM

We now combine the Eq. (1) and (2).we multiplying (2) by i and resultant to Eq. (1) to obtain

$$\frac{\partial q}{\partial t} + \left(2i\Omega + M^2 + \frac{1}{K}\right)q = \frac{\partial^2 q}{\partial z^2} + Gr\theta + G_C C \quad \ldots (11)$$

q=u+iv and

$$i = \sqrt{-1}$$

Further, for the radiative heat flux in Eq. (9) we invoke the differential approximation.

$$\nabla . q_r = 4(T - T_w)\int_0^\infty \alpha^2 \left(\frac{\partial B}{\partial T}\right) d\lambda \quad \ldots (12)$$

The Roseland approximation of Eq.(12) where the radiative flux vector q_r is given by

$$q_r = -\frac{4\sigma^*}{3\alpha}\frac{\partial T^4}{\partial z} \quad \ldots (13)$$

Assuming small temperature differences within the flow we can expand T in a Taylor series about a free stream temperature T neglecting higher order term, we obtain

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad \ldots (14)$$

Substituting Eq.(14) into Eq.(9) we obtain

$$\frac{\partial \theta}{\partial t} = \left(\frac{1+N}{\Pr}\right)\frac{\partial^2 \theta}{\partial z^2} \quad \ldots (15)$$

$$t \le 0 : q(z,t) = 0, \theta(z,t) = 0, C(z,t) = 0$$

$$t > 0 : \{\ q(0,t) = q_0, \theta(0,t) = 1 + \varepsilon \cos \omega t, C(0,t) = 1$$

$$\{\ q(\infty,t) \to 0, \theta(\infty,t) \to 0, C(\infty,t) \to 0 \quad \ldots (16)$$

$$C(z,t)=\frac{1}{2}\left\{e^{z\sqrt{k_r s_c}}erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}+\sqrt{k_r t}\right)+e^{-z\sqrt{k_r s_c}}erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}-\sqrt{k_r t}\right)\right\}$$

$$\theta=\left(erfc\frac{z}{2}\sqrt{\frac{a}{t}}\right)+\frac{\varepsilon}{4}\left[e^{-i\omega t}\left\{e^{-z\sqrt{-ia\omega}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}-\sqrt{-i\omega t}\right)+e^{z\sqrt{-ia\omega}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}+\sqrt{-i\omega t}\right)\right\}\right]$$

$$\frac{\varepsilon}{4}e^{i\omega t}\left[e^{-z\sqrt{ia\omega}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}-\sqrt{i\omega t}\right)+e^{z\sqrt{ia\omega}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}+\sqrt{i\omega t}\right)\right] \quad \ldots (17)$$

We now substitute Eq. (16) and (17) into Eq. (11) and take the inverse Laplace transform of the resultant equation. On appeal to Abramowitz and Stegun [20] for inverse transforms and convolution, we obtained

$$\left(\frac{q_0}{2}-\frac{a_2}{2a_5}-\frac{a_1}{2a_4}\right)\left[e^{z\sqrt{a_3}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{a_3 t}\right)+e^{-z\sqrt{a_3}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{a_3 t}\right)\right]$$

$$+\frac{a_2}{2a_5}e^{a_5 t}\left[e^{z\sqrt{a_3+a_5}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(a_3+a_5)t}\right)+e^{-z\sqrt{a_3+a_5}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(a_3+a_5)t}\right)\right]$$

$$+a_4 t\left(\frac{a_2}{2a_4}+\frac{a_1 a_8\varepsilon}{2}\right)\left[e^{z\sqrt{a_3+a_4}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(a_3+a_4)t}\right)+e^{-z\sqrt{a_3+a_4}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(a_3+a_4)t}\right)\right]$$

$$-\frac{a_1 a_6\varepsilon}{2}e^{-i\omega t}\left[e^{z\sqrt{a_3-i\omega}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(a_3-i\omega)t}\right)+e^{-z\sqrt{a_3-i\omega}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(a_3-i\omega)t}\right)\right]$$

$$+\frac{a_1 a_7\varepsilon}{2}e^{i\omega t}\left[e^{z\sqrt{a_3+i\omega}}erfc\left(\frac{z}{2\sqrt{t}}+\sqrt{(a_3+i\omega)t}\right)+e^{-z\sqrt{a_3-i\omega}}erfc\left(\frac{z}{2\sqrt{t}}-\sqrt{(a_3+i\omega)t}\right)\right]$$

$$-\frac{a_2}{2a_5}\left[e^{a_5 t}\left\{e^{z\sqrt{(k_r+a_5)Sc}}erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}+\sqrt{(k_r+a_5)t}\right)+e^{-z\sqrt{(k_r+a_5)Sc}}erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}-\sqrt{(k_r+a_5)t}\right)\right\}\right.$$

$$\left.-e^{z\sqrt{k_r Sc}}erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}+\sqrt{k_r t}\right)-e^{-z\sqrt{k_r Sc}}erfc\left(\frac{z}{2}\sqrt{\frac{Sc}{t}}-\sqrt{k_r t}\right)\right]$$

$$-\frac{a_1 e^{a_4 t}}{2}\left(\frac{1}{a_4}+a_8\varepsilon\right)\left[e^{z\sqrt{aa_4}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}+\sqrt{(a_4)t}\right)+e^{-z\sqrt{aa_4}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}-\sqrt{(a_4)t}\right)\right]$$

$$+\frac{a_1}{a_4}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}\right)+\frac{a_1 a_6\varepsilon}{2}e^{-i\omega t}\left[e^{-iz\sqrt{ai\omega}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}-i\sqrt{i\omega t}\right)+e^{iz\sqrt{ai\omega}}erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}}+i\sqrt{i\omega t}\right)\right]$$

$$-\frac{a_1 a_7 \varepsilon}{2} e^{i\omega t}\left[e^{z\sqrt{ai\omega}} erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}} + \sqrt{(ai\omega)t} \right) + e^{-z\sqrt{ai\omega}} erfc\left(\frac{z}{2}\sqrt{\frac{a}{t}} - \sqrt{(i\omega)t} \right)\right]$$

Where

$$a = \frac{\Pr}{1+N}, a_1 = \frac{Gr}{a-1}, a_2 = \frac{Gc}{Sc-1}, a_3 = 2i\Omega + M^2 + \frac{1}{K}, a_4 = \frac{a_3}{a-1}$$

$$a_5 = \frac{a_3 - Sck_r}{Sc-1}, a_6 = \frac{1}{2(i\omega + a_4)}, a_7 = \frac{1}{2(i\omega - a_4)}, a_8 = \frac{a_4}{a_4^2 + \omega^2}$$

IV. RESULT AND DISCUSSION

Numerical evaluation of analytical results reported in the previous section was performed and representative set of results is reported graphically in Figs.1-10. These results are obtained to illustrate the influence of various parameter on temperature (θ), concentration (C) and complex velocity (q), keeping the other parameter fixed. The solution obtained for the velocity is complex and only the real part of the complex quantity is invoked for the numerical discussion with the help of Abramowitz and Stegun [20].

Figure 1-7, shows the velocity profile for various parameters, taking ωt=π/2 and other parameters are fixed. The influence of the Grashof number Gr on the velocity profiles is illustrated in figure (1). It is evident that velocity increases with the increasing Grashof number. Physically it means that the buoyancy forces enhance the flow velocity. From Figure 2, it can be observed that the velocity decreases with the increase of magnetic field parameter (M). Physically, it means that the increasing M, the strength of the magnetic field, the Lorentz force increases which drag the flow backward. The velocity variations due to the Prandtl number (Pr) and permeability parameter (K) are given in figure (3). It is observed that the increase in Prandtl number decreases velocity. It is also observed that the velocity goes on increasing as the permeability increases. Physically this means that with the increasing permeability of the porous medium the resistance posed by the porous matrix goes on decreasing which consequently leads to the gain in the velocity. From fig.4, it is observed that the velocity decreases with increasing value of Sc. Furthermore from Fig.5 and 6, it is observed that velocity increases with increasing value of radiation parameter (N) and time parameter (t). Figure 7 represents the temperature profile for Pr=.71 (air) and Pr=7 (water). It is observed that temperature decreases as Pr increases. This is due to the fact that thermal conductivity of fluid decreases with increasing Pr. Further, we noticed that temperature increases with increasing value of radiation parameter (N). Concentration profiles are given in Fig. 9 and 10. We noticed that effect of increasing value of Sc is to decrease the concentration profiles. This is consistent with the fact that increase in Sc means decrease of molecular diffusivity D, which results in decrease of concentration boundary layer. Fig. (10) represents the concentration profiles for various values of chemical reaction parameter (Kr). It is observed that concentration profiles decreases as chemical reaction parameter (Kr) increase

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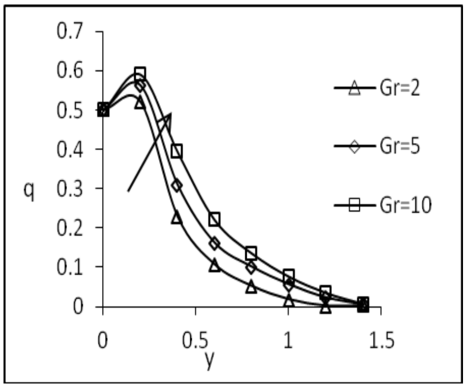


Fig.1 Effect of Grashof number (Gr) on velocity

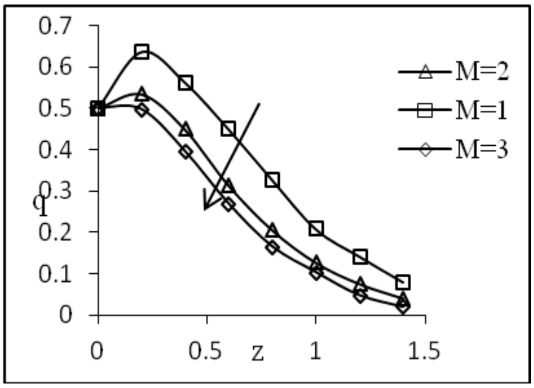
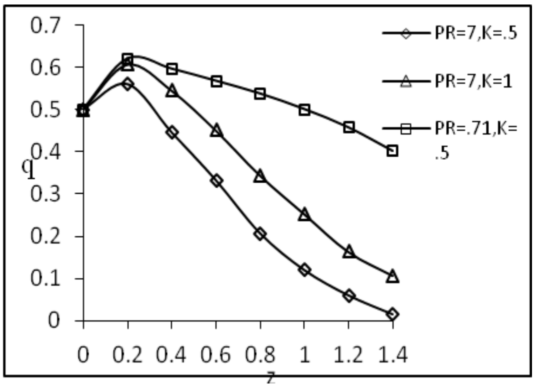


Fig.2 Effect of Hartman number (Gr) on velocity



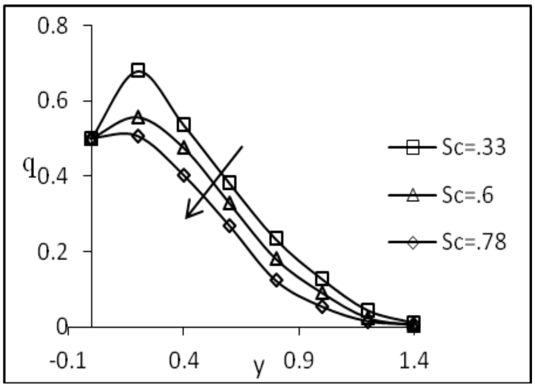


Fig.4 Effect of Schmidt number (Sc) on velocity

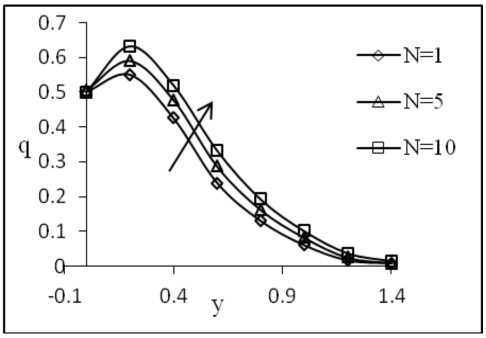


Fig.5 Effect of Radiation parameter number (N) on velocity

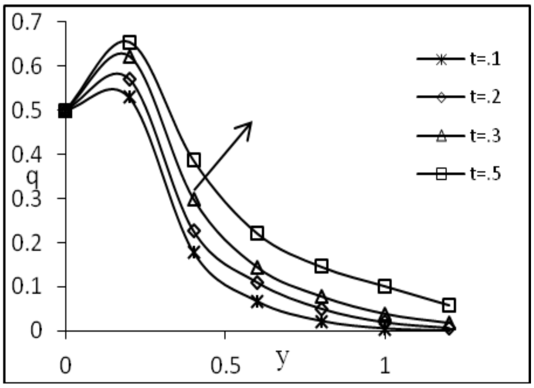


Fig.6 Effect of time parameter number (t) on velocity

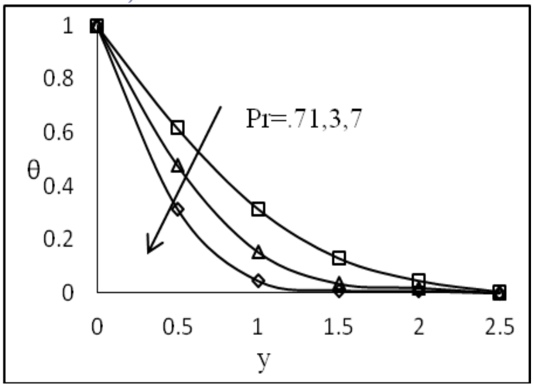


Fig.7 Effect of Prandtl number (Pr) on temperature

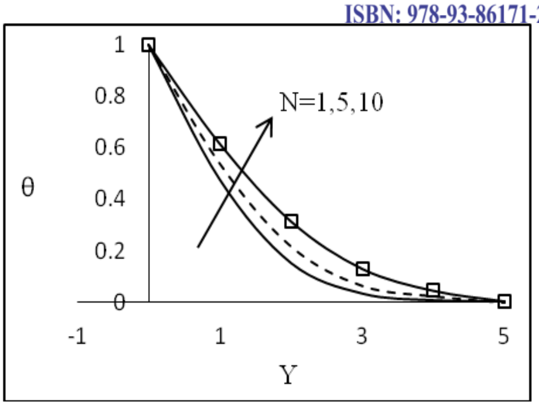


Fig.8 Effect of Radiation parameter (N) on temperature

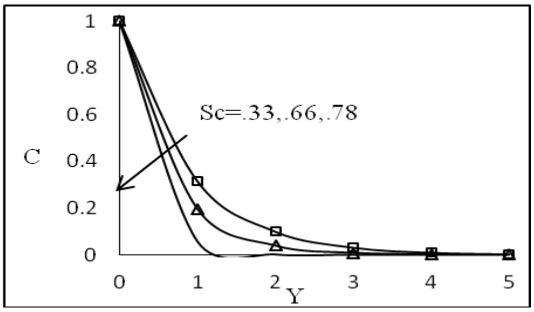


Fig.9 Effect of Schmidt number (Sc) on concentration

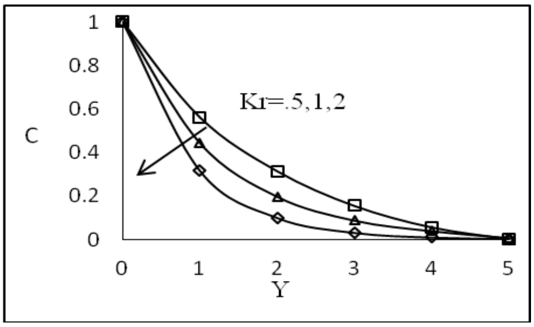


Fig.10 Effect of Chemical reaction parameter (Kr) on concentration

V. CONCLUSION

In the above study we find the effects of magnetic field and radiation on the fluid flow where the plate is oscillating with time. Concentration species show the effect of Schmidt number .it is observed that due to magnetic field the velocity decreases. Temperature increases due to radiation effect. Laplace transform technique is used to solve the equation in closed form.

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