

Efficiency of Black-Scholes Model for Pricing NSE INDEX Nifty50 Put Options and Observed Negative Cost of Carry Problem

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ABSTRACT

The Black-Scholes option pricing model is known for mispricing options on several parameters. Its result, applicability and causes have been tested by several researchers. This research paper empirically investigates the pricing accuracy of 2826 Put option contracts written over the underlying equity INDEX Nifty50 calculated under the Black-Scholes option pricing model. It has been observed that price of equity INDEX Nifty50 Put options contracts are overall undervalued by the Black-Scholes Model. Similarly, situation of overvaluation caused by the model in the case of ITM, OTM, Near Month, Next Month and Far Month contracts has been observed by the researchers in the valuation of equity INDEX Nifty50 put options . It has been observed that 24.87% of Nifty50 futures prices were quoted below their corresponding spot prices ignoring the concept of the cost of carry model. Hence, it shows that they are suffering from the negative cost of carry problem. The negative cost of carry problem has been addressed when the futures prices have been discounted at the prevailing risk- free rate. It has been found that 77.31% of Nifty50 futures prices were lower than their corresponding spot prices when they have been discounted.

Key Words: *Black-Scholes model, cost of carry, Discounted value, Far Month , In The Money, Near Month, Next Month and On The Money.*

I. INTRODUCTION

The aim of this research is to compare the INDEX Nifty50 futures prices to corresponding spot prices to gauge the existence of negative cost of carry bias. It has been seen that the prices of equity INDEX Nifty50 futures have been trading below their corresponding spot prices since its introduction on NSE's Derivative segment and ignores the application of cost of carry model. Further, to address the negative cost carry bias the DVFP has been compared to their corresponding spot prices. At the third stage the performance of the BS model at predicting INDEX Nifty50 put option prices traded in the Indian derivative market have evaluated.

A revolutionary change came in field of financial Derivatives when Fischer Black and Mayron Scholes' formula, known as the Black-Scholes model got published. They had published their paper in 1973 titled "The pricing of option and corporate Liabilities" in the Journal of Political Economy. The financial derivative tools

are particularly developed for minimizing the impact of associated risks. Hence, Black-Scholes option pricing theory is used to price financial derivatives and develop hedging strategies to minimize the impact of risks written on European-style options.

Trading in derivative products is one of the most important principal opportunities of an effective securities market. The Black-Scholes option pricing model is a landmark in the history of Financial Derivative. This preferred model provides a closed analytical view for the valuation of European-style options. An option is a standardized financial contract, which gives the buyer the right, but not the obligation, to buy or sell specified quantity of the underlying assets, at a strike price on or before the expiration date. The underlying may be physical commodities like crude oil, wheat, rice, cotton etc. or financial instruments like equity stocks, equity index, bonds etc. There are two types of option- call and put option. The call option gives the buyer the right but not the obligation to buy whereas the put option gives the right but not the obligation to sell the underlying. Option allows people to bet on the future events and to reduce the associated financial risk. There are two kinds of options- American options and European options. The former may be exercised any time before its expiration date while the later can be exercised on its expiration date. The stock option contracts are priced by the Black-Scholes model which exhibits pricing errors on several occasions.

1.1 BLACK-SCHOLES MODEL

Based on the above mentioned assumptions, Fisher Black and Myron Scholes have developed the following equation for the valuation option for no dividend paying stocks (Hull, 2007)-

$$P = X.e^{-rt}N(-d_2) - S.N(-d_1)$$

Where,

$$d_1 = \frac{\ln(S/X) + (r + 0.5\sigma^2) t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(S/X) + (r - 0.5\sigma^2) t}{\sigma\sqrt{t}}$$

The variables are-

P is the Put Price of stock option, S_0 is the Current Stock price, X the is Exercise price, T the is Time remaining until expiration, expressed as a percent of a year, r the is Current Continuously compounded risk-free interest rate and σ is the Standard deviation of stock's return.

The unknown parameter of this model is σ . The Black-Scholes Model says that the option price, no matter it is put or call, is a function of asset price, time to maturity, exercise price, compounded risk free interest rate and volatility of asset price. All those variables except for the volatility are easily obtainable from the market. σ is the only unknown factor in the formula. σ is assumed unchanged while calculating option prices. σ is calculated

through two approaches- historical volatility and implied volatility. The historical volatility is calculated by the annualized standard deviation of historical daily returns. The historical approach is much simpler than the other one. The implied volatility looks more on the future movements.

II. REVIEW OF LITERATURE

A number of researches have been carried out by the researchers to judge the pricing accuracy of the Black-Scholes model. Blattberg and Gonedes (1974) studied the impact of volatility, as one of the basic parameters on the option price provided by the Black-Scholes model and concluded that volatility of the underlying stock is stochastic and random. Black (1975) had also identified that this model suffers from the pricing errors.

Some researchers have also examined the affect of option maturity. Time to maturity has also effect on the calculation of option prices under the Black-Scholes model. MacBerth and Merville (1979), using implied volatility instead of historical volatility, find that implied volatility is high for in-the-money options but the Black-Scholes model underprices these in-the-money options and secondly implied volatility is low for the out-of-the-money options but this model considerably overprices these out-of-the-money options.

Rubinstein (1985) has examined the implied volatility on the 30 options classes (Chicago Board of Option Exchange) for a period of two years (from August 1976 to August 1978) and found that the short maturity options had higher implied volatility than long maturity options.

However, some researches show that discrepancies between the market option prices and prices calculated under the Black-Scholes model are not large enough to be exploited. LauterBach and Schultz (2012) on pricing warrants, Jordan and Seale (1986) and Blomeyer and Boyd (1988) on futures options written on treasury bond have suggested that there is a very little difference between the market actual price and the Black-Scholes predicted price. Bailey (1987) studied on future option written on gold, Shastri and Tandon (1986) on Future (American options), and Jordan, Seale, McCabe and Kenyon (1987) on futures options written on soyabean have found discrepancies are not enough in the model predicted prices.

Varma (2003) has studied volatility, using data for a short period of time from June 2001 to February 2002, on Nifty Future and options prices under the Black-Scholes model. He suggests that the volatility is severely mispriced because of the imperfection of the Indian market and market is learning and the impact of learning effects can be seen over a long period of time.

Ramazan Gencay and Aslihan Salih (2003) compare the Black-Scholes model against the Feedforward Networks Model using S&P 500 option Index data from January 1998 to December 1993. They suggest that the Black-Scholes Model exhibits pricing error at several occasions especially for the deeper out-of-the money options compare to the near out-of-the-money options and this pricing error worsens with increased in volatility. Hence, Feedforward networks provide less pricing error as compare to the Black-Scholes model for the deeper out-of-money options.

Rinalini Pathak Kakati (2006) studied the effectiveness of Black-Scholes option pricing model in the Indian context using 2342 put and 1280 put options written from July 2001 to March 2003. She found that the Black-

Scholes model misprices options considerably on several occasions. Pricing errors are negative on an average and significantly different from zero. She further suggests that mispricing worsens with both increased in moneyness and increased in the volatility of the stocks. The Black-Scholes model, according to Rinalini Pathak Kakati (2006), overprices short-term options and underprices long-term options.

Mckenzie, Gerace and Sbedar (2007) have studied the pricing errors produced by the Black-Scholes model using ASX 200 option Index and suggested that the use of a jump-diffusion approach and implied volatility instead of historical volatility increases the tail properties of the underlying lognormal distribution. Consequently, it increases the pricing accuracy of the Black-Scholes model¹. Rubinstein (1994) studied extensively on implied volatility under the Black-Scholes model for S&P 500 option Index and states that the Black-Scholes model may under price options because the tail properties of underlying lognormal distribution are very small.

Emilia Vasile and Dan Armeanu (2009) have worked on the mispricing errors produced by the Black-Scholes Model for pricing options contracts. The operators take into consideration the moneyness of an option and the duration up to the due term thereof, when they calculate the volatility on account of which they evaluate the option. This is a direct consequence of the fact the Black-Scholes model cannot be applied in its original form: the prices of the financial assets do not follow log normal distribution.

Tripathi & Gupta (2010) examined the predictive accuracy of the Black-Scholes (BS) model in pricing the Nifty Index option contracts by examining whether the skewness and kurtosis adjusted BS model of Corrado and Su gives better results than the original BS model. They have also examined whether volatility smile in case of NSE Nifty options, if any, can be attributed to the non-normal skewness and kurtosis of stock returns. Based on data of S&P CNX NIFTY near-the-month put options for the period January 1, 2003 to December 24, 2008, their results show that BS model is misspecified as the implied volatility graph depicts the shape of a Smile" for the study period. There is significant under-pricing by the original BS model and that the mispricing increases as the moneyness increases. Even the modified BS model misprices options significantly. However, pricing errors are less in case of the modified BS model than in case of the original BS model. On the basis of Mean Absolute Error (MAE), they concluded that the modified BS model is performing better than the original BS model.

Subrata Kumar Mitra (2012) found that the total error in Black model was less than the Black-Scholes model. During his study, the mean error of the Black was 1.76 and the mean error of Black-Scholes was -10.49. The result was significant with the value of $p = .000$. He compared NIFTY Future Price with the theoretiputy calculated value that includes cost of carry and found that stock Index Futures sometime suffer from a negative cost of carry bias. He has not compared Future prices with the spot prices to see the deformities.

V. Panduranga (2013) using Banking sector stocks on testing the efficiency of the Black-Scholes Model found that the model is applicable for banking sector stocks in India. Paired sample T-test results indicate that this model can be applied for banking stock options. However, he has observed that in one out of four cases, there is a difference between expected price and market price of the option. Options may be under-priced or overpriced in the market.

Nagendran and Venketeswar (2014) have worked on 95,000 equity call options to test the validity of the Black-Scholes model in written on Indian National Stock Exchange. They found that there is an improvement in Black-Scholes model. Their results show the robustness of the Black-Scholes model in pricing stock options in India and that is further improved by incorporating implied volatility into the model.

2.1 Movements in the futures price: Kawaller, Koch and Koch (1988) had studied the Lead-leg relationship between the price movements of S&P 500 Index futures and S&P 500 Index traded over the New York Stock Exchange. Evidence uncovered in their tests of lagged relationship between the S&P 500 Index futures prices and S&P 500 Index pointed to the usefulness of the futures as a predictor of broad equity market movements measured by the Index. Their study empirically proved that the Index shown lags of up to 45 minutes behind the futures and hence this consistency implied that Index futures trading continued to make its contribution to price discovery.

Frank and Atle (2002) investigated the relationship between Futures price and spot prices using Johansen test, a multivariate framework used to test relationship between variables. They have tested relationship between prices movements on contract written on gas oil. Their findings indicate that futures price leads spot prices and that future contract written on gas oil with longer time to expiration leads contracts with shorter time to expiration.

Mukherjee and Mishra (2006) have examined lead-leg relationship on the intraday trading at NSE. They found that the spot market played a comparatively stronger leading role in disseminating information available to the market and therefore said to be more efficient. They further suggest that the results relating to the informational effect on the lead-lag relationship exhibit that though the leading role of the futures market wouldn't strengthen even for major market-wide information releases, the role of the futures market in the matter of price discovery tends to weaken and sometime disappears after the release of major firm-specific announcements.

III. COST OF CARRY

Spot and futures are linked by a cost of carry relationship and hence futures price may contribute to the discovery of new information (Lin and Stevenson (1999)). The relationship between future price and spot price can be summarized in the terms of the cost of carry. This measures the storage cost plus the interest that is paid to finance the asset less the income earned on the asset. Nationalized Bank interest rate on short-term fixed deposit as also T-bills yield of 90 days has been taken from RBI website www.rbi.org.in for the above said period. It is somehow difficult to calculate dividend yield for Nifty 50 as all the constituent stocks of Nifty do not pay dividend in one time or in one installment, some also pay interim dividend. For this purpose, dividend yield for Nifty 50 would be taken for the above mentioned period.

Hence, for an investment asset, if cost of carry is defined as 'c', the futures price is

$$F_0 = S_0 e^{ct}$$

{John C. Hull, (2007), "Options, Futures, and other Derivatives", Sixth Edition}

IV. OBJECTIVES OF THE STUDY

The primary objectives of this research paper are:

- a) To compare the INDEX Nifty50 futures prices with corresponding spot prices to identify the existence of negative cost of carry problem.
- b) To compare the INDEX Nifty50 Discounted value of futures prices with corresponding spot prices to address the negative cost of carry problem.
- c) To investigate the INDEX Nifty50 Put options pricing errors calculated under the Black-Scholes Model.
- d) To investigate the Money and Maturity biasness of INDEX Nifty50 Put options calculated under the Black-Scholes Model.

4.1 RESEARCH ASSUMPTIONS

- a) Research paper has tested the equity INDEX Nifty50 Put options and futures prices quoted in the Indian National Stock Exchange (NSE) only.
- b) The historical Nifty50 Put option prices are considered as the closing prices.
- c) The Nifty50 Put option prices used are reliable and accurate.

V. DATA COMPILATION AND ANALYSIS

This research paper examines 2826 put option contracts written on underlying INDEX Nifty50 during 2008 and 2010. For this purpose sample consists of closing prices of 2826 Put options (observations), written on underlying INDEX Nifty50 of Indian National Stock Exchange, have been collected from the website of exchange www.nseindia.com. To bring the uniformity among the data and to reduce the effect of the time gap in closing prices, this research paper considers only highly traded options and which are expected to trade until the last moment on the concern stock exchange.

Table 1- Year wise No. of observation

| Observations/Year | Jun.2008- May 09 | Jun.2009- May 10 | Jun.2010- May 11 | Jun.2011- May 12 | Total No. of Observations |
|---------------------|---------------------|---------------------|---------------------|---------------------|------------------------------|
| No. of Observations | 678 | 706 | 726 | 716 | 2826 |

(b) Types of Data: Secondary data of Put option have been collected and used for the purpose of the calculation of the theoretical predicted premium prices as well as for the standard deviation of the stock option.

(c) Period: This research paper examines 2826 Put option prices written on underlying INDEX Nifty50 for a period of 4 years (from Jun. 2008 to May. 2012).

(d) Sources of data: The primary sources of data, time, contract month, option types, strike price and closing prices etc. are, for the purpose of this study paper have been taken from the Indian Stock Exchange website www.nseindia.com (assessed from Jun. 2008 to May. 2012). The risk free interest rate used in this analysis is

the 90 days T. Bills of RBI (Reserve Bank of India) website www.rbi.org.in (assessed from Jun. 2008 to May. 2012).

(e) Processing of data: The collected data have been entered under the data base and processed under the Microsoft Excel for the final calculation of the standard deviation and theoretical prices of the stock option under the Black-Scholes model. The calculated prices of put options are also cross checked by the software FUTOP version 2.0.0 used for the calculation of option prices.

(f) Time to expiry: In the above mentioned formula term 't' is the time left for the option to expire. Here, calendar days has been used to calculate 't', irrespective of intervening holidays. Further, 't' is annualized by dividing 't' by 365 days.

(g) Volatility: Volatility is a measure of the uncertainty on the return provided by the INDEX Nifty50. Here in the Black-Scholes model Volatility means standard deviation of the continuous compounded return. In this research researchers have used Hull's suggested formula. Hull (2004, page no. 263) has suggested following formula to calculate annual volatility-

$$\text{Volatility Per Annum} = \text{Volatility per trading day} \times \sqrt{\text{No. of trading days per annum}}$$

(h) The pricing accuracy- The pricing accuracy of stock options provided by the Black-Scholes are compared by the Mean Error (ME), Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Thiel's Inequality Coefficient:

(i) Mean Error (ME): ME is calculated by adding all error and dividing total error by the number of observations-

$$\text{Mean Error} = \frac{1}{N} \sum_{N=1}^N e_n$$

The result is acceptable when the all error data have the same sign (either all are positive or all are negative).

(ii) Mean Absolute Error (MAE)

The mean absolute error value is the average absolute error value. The closer this value is to zero, the better is the forecast. The neutralization of positive errors by negative errors can be avoided in this measure. MAE is computed using the formula-

$$\text{MAE} = \frac{1}{N} \sum_{N=1}^n |e_n|$$

(iii) Mean Squared Error (MSE): Mean squared error is computed as the average of the squared error values. This is the commonly used error indicator in statistical fitting procedures. As compare to the mean absolute error value, this measure is very sensitive to large outlier as it places more penalties on large errors than mean absolute error. MSE is computed using the following formula-

$$MSE = \frac{1}{N} \sum_{n=1}^n e_n^2$$

(iv) Root Mean Squared Error (RMSE):

It is the square root of mean squared error and conceptually similar to the widely used statistic called- Standard Deviation-

$$RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^n e_n^2}$$

(v) Thiel's U statistic (Inequality coefficient):

Henri Theil (1961) developed an inequality coefficient for measuring the degree to which one time series differs from another. Theil's U statistic is computed as under-

$$U = \frac{\sqrt{\frac{1}{N} \sum_{n=1}^N (y_n - f_n)^2}}{\sqrt{\frac{1}{N} \sum_{n=1}^N y_n^2} + \sqrt{\frac{1}{N} \sum_{n=1}^N f_n^2}}$$

Here, the two time series in question are (a) the actual value of options (y_n) and (b) the value of option predicted by the models (f_n). Thiel's U will equal 1 if a forecasting method is found no better than using a naive forecast. If Thiel's U is less than 1 it indicates that the method is superior to a naive forecast. A value close to zero indicates a good fit, whereas, value greater than 1 indicates that the technique is actually worse than using a naive forecast.

VI.FINDING AND DISCUSSION

Table 2- comparison of futures prices with corresponding spot prices

| Year | Total No. of Observations | No. of observations when FP<SP | No. of observations when FP>SP | No. of observations when FP<SP in %age |
|----------------|---------------------------|--------------------------------|--------------------------------|--|
| Jun.08-May. 09 | 678 | 323 | 355 | 47.64 |
| Jun.09-May. 10 | 706 | 216 | 490 | 30.59 |
| Jun.10-May. 11 | 726 | 94 | 632 | 12.94 |
| Jun.11-May. 12 | 716 | 70 | 646 | 9.77 |
| Total | 2826 | 703 | 2123 | 24.87 |

It has been observed that 24.87%, of Nifty50 futures prices, 703 out of 2826 observation, were quoted, as stated in table 2, below their corresponding spot prices.

Table 3- comparison of Discounting Value of futures prices with corresponding spot prices

| Year | Total No. of Observations | No. of observations when DVFP<SP | No. of observations when DVFP>SP | No. of observations when DVFP<SP in %age |
|----------------|---------------------------|----------------------------------|----------------------------------|--|
| Jun.08-May. 09 | 678 | 526 | 152 | 77.58 |
| Jun.09-May. 10 | 706 | 582 | 124 | 82.43 |
| Jun.10-May. 11 | 726 | 536 | 190 | 73.82 |
| Jun.11-May. 12 | 716 | 541 | 175 | 75.55 |
| Total | 2826 | 2185 | 641 | 77.31 |

The Discounting Value of Futures Prices (DVFP) has been compared with their corresponding Spot Prices (SP) in table 3. It has been found that 77.31% of Nifty50 futures prices, 2185 out of total 2826 observations, were lower than their corresponding spot prices when they have been discounted. Hence, 77.31% of the observations are likely to be get affected by the negative cost of carry problem.

Table 4- Paired Sample t-test for Pricing Nifty50 Put options

| | Paired Differences | | | | t | df | Sig.(2-tailed) | |
|-------------------------------------|--------------------|----------------|-----------------|---|-------|-------|----------------|-------|
| | Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference | | | | |
| | | | | Lower | | | | Upper |
| Pair 1 Closing price-BS Model Price | 18.86 | 86.66 | 1.63 | 15.67 | 22.06 | 11.57 | 2825 | .000 |

The p value of SPSS output is lower than 0.05, as stated in table 4, which causes us to reject null hypothesis. The t-statistic is greater than critical value. Hence, there is a significance difference between the market put close prices and prices calculated under the B&S Model for put options written on INDEX Nifty50.

Table 5- Errors of Black-Scholes Model for Pricing Nifty50 Put options

| Total No. of observations | Mean Error (ME) | Mean Absolute Error (MAE) | Mean Squared Error (MSE) | Root Mean Squared Error (RMSE) | Thiel's U statistic |
|---------------------------|-----------------|---------------------------|--------------------------|--------------------------------|---------------------|
| 2826 | 18.86 | 35.26 | 7858.19 | 88.65 | 0.0975 |

The overall pricing accuracy of the B&S Model has been also tested on the parameters of ME, MAE, MSE, RMSE and Thiel's U Statistic to bring the clarity in research. Here, the mean pricing bias is not zero in every case as shown in Table 5. Therefore, the null hypothesis is being rejected. The mean bias differs significantly from zero. The Black-Scholes model, hence, exhibits pricing errors.

Table 6- Moneyness bias of Black-Scholes Model for Pricing Nifty50 Put options

| Moneyness | No. of observations | ME | MAE | MSE | RMSE | Thiel's U statistic |
|-----------|---------------------|-------|-------|---------|-------|---------------------|
| ITM | 1065 | 12.72 | 35.98 | 8355.69 | 91.41 | 0.0626 |
| OTM | 1761 | 22.57 | 34.82 | 7557.32 | 86.93 | 0.4443 |

An option may be ITM, ATM or OTM from its moneyness point of view. The Moneyness accuracy of the B&S Model has been tested in Table 6. The mean pricing biasness for moneyness (for both ITM and OTM) is not zero. Hence, model shows pricing errors in the calculation of ITM and OTM options. However, as compare to pricing error of ITM option, the magnitude of misfit is very high in the case of OTM option pricing error (0.4443) on the basis of Theil's U statistic. The ATM options have not been found during our stated study period.

Table 7- Maturity bias of Black-Scholes Model for Pricing Nifty50 Put options

| Moneyness | No. of observations | ME | MAE | MSE | RMSE | Thiel's U statistic |
|------------|---------------------|-------|-------|----------|--------|---------------------|
| Near Month | 862 | 7.16 | 26.02 | 15558.46 | 124.73 | 0.1221 |
| Next Month | 971 | 19.21 | 27.37 | 1463.89 | 38.26 | 0.0387 |
| Far Month | 993 | 28.67 | 50.99 | 7426.40 | 86.17 | 0.1226 |

Table 7 shows about the pricing errors of different lives of put option, i.e., near month, next month and far month. The mean pricing biasness for different maturities is not zero. Hence, the B&S model exhibits pricing errors in the calculation of near month, next month and far month Nifty50 put options' price.

VII. CONCLUSION

It has been observed that, 24.87%, majority of time futures prices of INDEX Nifty50 were traded below their corresponding spot prices which shows that futures prices are suffering from negative cost of carry bias and it became 77.31% when the futures prices of INDEX Nifty50 have been discounted. Hence, The Black-Scholes Model exhibits pricing errors on several parameters in calculation put options written on 2826 equity INDEX Nifty50 of Indian National Stock Exchange. The significant put options pricing errors for ITM, OTM, Near month, Next month and Far month have been observed in the Indian Derivatives market. It has been found that the INDEX Nifty50 put options are severely mispriced by the BS model due to negative cost of carry bias. and compare with the performance of the model after replacing spot price (S) by the DVFP in the BS model. Minor as well as major improvements have been found on the various parameters used to calculate errors when the spot price is being replaced by DVFP in the original BS model.

VIII. SCOPE FOR FURURE STUDY

The Black-Scholes model suffers from the pricing errors for the calculation of INDEX Nifty50 put options prices. Since the inception of Nifty future in India, it has been observed that the Nifty50 future have been trading even below the corresponding Nifty50 spot price. Hence, this model suffers from the cost of carry

problem. Pricing Errors can be minimized, if the negative cost of carry problem is addressed by replacing the spot price (SP) by the Discounting Value of Futures Prices (DVFP) in the original Black-Scholes model.

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