A SIMPLEPOLE CLUSTERING TECHNIQUE FOR DYNAMIC SYSTEMS SIMPLIFICATION

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ABSTRACT

A simple clustering technique is presented in this paper to simplify the dynamic system via reducing the order of its transfer function. The reduced denominator polynomial of thesimplified model is determined by a simple pole clustering technique and the numerator coefficients are obtained byusing improved Pade approximations method. The dominant pole of a cluster may be inside or outside of the pole cluster unlike modified pole clustering technique already suggested in the literature. The proposed method is computationally simple, efficient and can be easily programmed. The method is capable to retain transient, steady-state value and stability features of the original large-scale dynamic system. The viability of the proposed method has been tested on few high order systems.

Keywords : Clustering technique, Integral square error,Order reduction, Stability,System simplification

I. INTRODUCTION

The simplification of the systems via reducing order of the transfer function is an important research area in control applications. Whenever, a dynamic model of any physical system is required to be analyzed, it is very often noticed that the dynamic model may be complex in nature. The approximation or reduction of a complex model is also known as simplification of the system.The complexity of the dynamic systems often result a difficulty to understand the behaviour of the system. The preliminary design and optimization of such systems can be done effectively and with greater ease, if a simplified model is synthesized from the original high- order system.

The varieties of simplification method [1-4] for the linear systems have been suggested and are available in the literature. Few authors [5-8] have already suggested pole clustering techniques in which suitable number of pole clusters are formed and using algorithm a cluster centre is found, which is treated as dominant pole of that cluster. Unlike the previous papers based on clustering, the proposed algorithmhas shown that cluster centre may be outside the cluster depending upon the value and number of the poles in the cluster.In this algorithm, it is shown that the simplified models obtained are better than previous algorithms based on pole clustering.

II. PROBLEM FORMULATION

Consider a large –scale dynamic system of the order $'n'$ as

$$G(s)=\frac{a_0+a_1 s+a_2 s^n+\ldots+a_{n-1}s^{n-1}}{b_0+b_1 s+b_2 s^2+\ldots+b_n s^n} \qquad (1)$$

Let corresponding k^{th} -order simplified model be represented as

$$R_k(s)=\frac{N_k(s)}{D_k(s)}=\frac{c_0+c_1 s+c_2 s^2+\ldots+c_{k-1}s^{k-1}}{d_0+d_1 s+d_2 s^2+\ldots+d_k s^k} \qquad (2)$$

The problem is to obtain simplified model $R_k(s)$ from the original system $G(s)$ using the proposed algorithm.

III. DESCRPTION OF THE PROPOSED METHOD

The proposed algorithm may be described as under:

STEP-1: Denominator polynomial of the simplified model by using proposed method which is slight modification of the paper [7].

Suppose the i^{th} - pole cluster consists of the poles as $|p_1|<|p_2|<|p_3|\ldots..<|p_r|$.

Then pole cluster centre p_{ci} is obtained as

$$p_{ci}=\left(p_1+p_2+\ldots+p_r\right)\div 2r \qquad (3)$$

Similarly, if a cluster contains $'m'$ number of complex poles such as $[(\sigma_1\pm j\lambda_1),(\sigma_2\pm j\lambda_2)\ldots..(\sigma_m\pm j\lambda_m)]$, then using proposed algorithm, cluster centre can be obtained as $A_c\pm jB_c$.

Where $A_c=\left\{\left(\sum_{i=1}^{m}(\sigma_i)\right)\div 2m\right\}$ and $B_c=\left\{\left(\sum_{i=1}^{m}(\lambda_i)\right)\div 2m\right\}$ (4)

The reduced denominator $D_k(s)$ of the simplified model can be obtained from any one of the given cases:

Case-1 For all real pole cluster centres, the denominator of reduced model is written as

$$D_k(s)=(s-p_{c1})(s-p_{c2})\ldots(s-p_{ck}) \quad (5)$$

Case-2 if $(k-2)$pole cluster centres be real and one complex cluster centre

$$D_k(s)=(s-p_{c1})(s-p_{c2})\ldots(s-p_{c(k-2)})(s-\overset{0}{p}_{c1})(s-\overset{*}{p}_{c1})) \qquad (6)$$

Where $\overset{0}{p}_{c1}$ and $\overset{*}{p}_{c1}$ are complex cluster centres i.e. $\overset{0}{p}_{c1}=A_c+jB_c$ and $\overset{*}{p}_{c1}=A_c-jB_c$

Case-3 For all complex pole cluster centres are, the reduced denominator polynomial is obtained as

$$D_k(s)=(s-\overset{0}{p}_{c1})(s-\overset{*}{p}_{c1})(s-\overset{0}{p}_{c2})(s-\overset{*}{p}_{c2})\ldots(s-\overset{0}{p}_{ck/2})(s-\overset{*}{p}_{ck/2}) \quad (7)$$

STEP-2: Reduced numerator coefficients by improved Pade approximations [9].

$$G(s)=\frac{a_0+a_1s+\ldots+a_{n-1}s^{n-1}}{b_0+b_1s+\ldots+b_ns^n}$$

$$=\sum_{i=0}^{\infty}M_is^{-i-1} \qquad \text{(about } s=\infty\text{)(8)}$$

$$=\sum_{i=0}^{\infty}T_is^{i} \qquad \text{(about } s=0\text{)(9)}$$

The coefficients of $N_k(s)$ can be obtained as

$$\left.\begin{aligned}
c_0&=d_0T_0\\
c_1&=d_0T_1+d_1T_0\\
&\ldots\\
&\ldots\\
c_{\alpha-1}&=d_0T_{\alpha-1}+d_1T_{\alpha-2}+\ldots+d_{\alpha-2}T_1+d_{\alpha-1}T_{0c}\\
c_{k-\beta}&=d_kM_{\beta-1}+d_{k-1}M_{\beta-2}+\ldots+d_{k-\beta+2}M_1+d_{k-\beta+1}M_0\\
&\ldots\\
&\ldots\\
c_{k-2}&=d_kM_1+d_{k-1}M_0\\
c_{k-1}&=d_kM_0
\end{aligned}\right\} \quad (10)$$

With $\alpha+\beta=k$

Hence, numerator $N_k(s)$ can be obtained as

$$N_k(s)=c_0+c_1s+c_2s^2+\ldots+c_{k-1}s^{k-1} \quad (11)$$

IV. RESULTS AND COMPARISON

Two high order systems are taken from literature and using the proposed method lower order simplified models are obtained. The simplified models are compared with the large-scale systems graphically and analytically. The performance index Integral Square Error (ISE) [10] is computed between the original systems and reduced simplified models. The ISE is defined as

$$\text{ISE}=\int_0^{\infty}[x(t)-x_k(t)]^2\,dt \quad (12)$$

Where $x(t)$ and $x_k(t)$ are unit step function of original and reduced simplified model respectively.

Example-1: An eight order large-scale system is taken from literature [11]

$$G(s)=\frac{18s^7+514s^6+5982s^5+36380s^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109584s+40320}$$

The poles are: -1, -2, -3, -4, -5, -6, -7,-8

Two clusters may be made as (-1,-2,-3,-4) and (-5,-6,-7,-8) are considered. The corresponding pole cluster centres are computed as

p_{c1} = -1.25 and p_{c2} = -3.25

Hence, $s^2+4.5s+4.0625$

Using Step-2, the time moments and Markov parameters are

$T_0=1 \quad T_1=1.8904 \quad T_2=-2.5592$
$M_0=18 \quad M_1=-133.952 \quad M_2=975.872$

Two numerators are possible with this algorithm as

If $\alpha=2 \quad \beta=0$

$c_0=d_0T_0=4.0625$

$c_1=d_0T_1+d_1T_0=12.1797$

$N_{21}(s)=12.1797s+4.0625$

Similarly, $\alpha=1 \quad \beta=1$

$c_0=d_0T_0$=4.0625

$c_1=d_2M_0$=1 ×18=18

$N_{22}(s)=18s+4.0625$

$$R_{21}(s)=\frac{N_{21}(s)}{D_2(s)}=\frac{12.1797s+4.0625}{s^2+4.5s+4.0625} \text{ and } R_{22}(s)=\frac{N_{22}(s)}{D_2(s)}=\frac{18s+4.0625}{s^2+4.5s+4.0625}$$

Similarly, 3rd –order models are obtained with the following clusters and its centres as

Cluster-1: (-1, -2, -3) , $p_{c1}=-1$

Cluster-2: (-4, -5, -6), $p_{c2}=-2.5$

Cluster-3: (-7, -8), $p_{c3}=-3.75$

Hence
$$\begin{aligned} D_3(s) &= (s+1)(s+2.5)(s+3.75) \\ &= s^3+7.25s^2+15.625s+9.375 \end{aligned}$$

$$R_{31}(s)=\frac{12.7855s^2+33.3475s+9.375}{s^3+7.25s^2+15.625s+9.375} \text{ for } (\alpha=3,\beta=0)$$

$$R_{32}(s)=\frac{18s^2+33.3475s+9.375}{s^3+7.25s^2+15.625s+9.375} \text{ for } (\alpha=2,\beta=1)$$

$$R_{32}(s)=\frac{18s^2-3.452s+9.375}{s^3+7.25s^2+15.625s+9.375} \text{ for } (\alpha=1,\beta=2)$$

The 2nd and 3rd –order simplified models are shown in the Fig-1 and the ISE is calculated and compared with the available methods and tabulated in Table-1. The 3rd –order simplified model ($\alpha=3,\beta=0$) has ISE 0.01952.

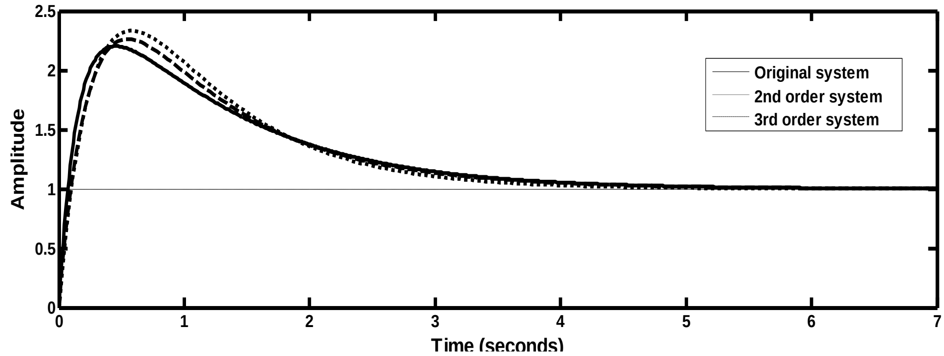

Figure-1 Step Response Comparison for example-1

Table-1: Comparison of proposed method with well-known methods

Simplification Methods	**Reduced Models**	**ISE**
Proposed method	$R_{21}(s)=\dfrac{12.1797s+4.0625}{s^2+4.5s+4.0625}$	0.04226
	$R_{22}(s)=\dfrac{18s+4.0625}{s^2+4.5s+4.0625}$	1.127
C.B. Vishwakarma [7]	$\dfrac{16.511s+5.459}{s^2+6.196s+5.459}$	0.0140
G. Parmar *et al.* [11]	$\dfrac{24.1142s+8}{s^2+9s+8}$	0.0480
Mittal *et al.* [12]	$\dfrac{7.0908s+1.9906}{s^2+3s+2}$	0.2689
Prasad and Pal [4]	$\dfrac{17.985s+500}{s^2+13.245s+500}$	1.4584

Example-2: Consider a 4th –order dynamic model [13] having its transfer function as

$$G(s)=\frac{28s^3+496s^2+1800s+2400}{2s^4+36s^3+204s^2+360s+240}$$

The poles of the system are $(-7.8033\pm j1.3576)$ and $(-1.1967\pm j0.6934)$

The few time moments and Markov parameters of system are

$T_0=10,\quad T_1=-7.5,\quad M_0=14$

Using the proposed method, the following 2nd -order simplified models are determined as

$$R_{21}(s)=\frac{4.85s+54.6}{s^2+4.58s+5.46}\qquad(\alpha=2,\beta=0)$$

$$R_{22}(s)=\frac{14s+54.6}{s^2+4.58s+5.46}\qquad(\alpha=1,\beta=1)$$

Both the 2nd –order models are graphically compared with the original 4th –order system and it has been seen that simplified models are very close to the original system.

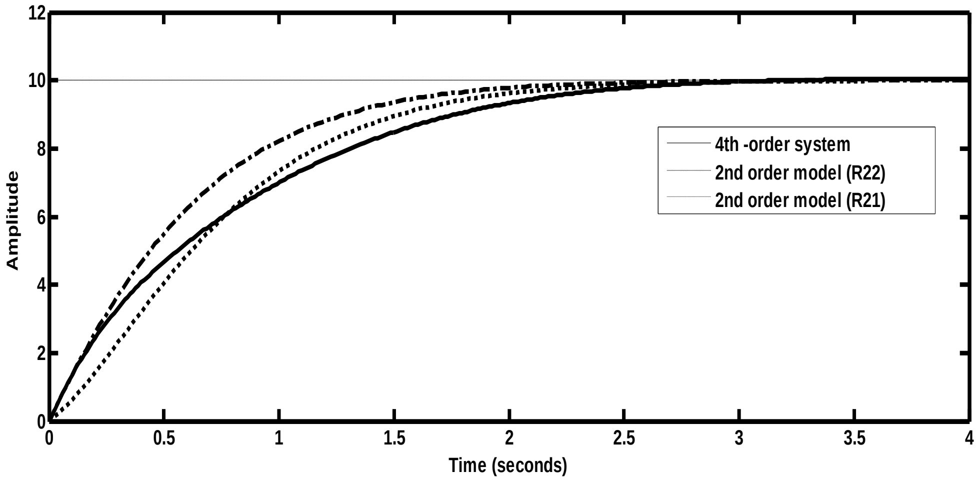

Figure-2 Step Response Comparison for example-2

V. CONCLUSION

A simple and new pole clustering technique for simplification of linear dynamic systems using improved Pade approximations is proposed. This method gives 'k' number of simplified models for k^{th} -order simplification. Two examples are taken from literature to show validity of the proposed method for system simplification. From both examples, it is clear that the simplified models are near to the original system and better than few well-known simplification methods and also capable to retain stability and final value of the original system. From table-1, it is very much clear that reduced model obtained by the proposed method has lesser error than few simplification methods. The reduced models closely matching the large-scale system in the both examples. The proposed method is easily extendable to multi-inputs multi-outputs system as well.

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