ISBN: 978-93-86171-74-0

Propagation of Waves at Loosely Interface Between Solid Media

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ABSTRACT

The present investigation deals with propagation of longitudinal wave from a loosely bonded interface separating micropolar elastic solid half space and micropolar viscoelastic solid half space with stretch. A longitudinal wave is considered to be incident on the plane interface through micropolar elastic solid half space. A longitudinal wave bumps into obliquely at the interface. Amplitude ratios of various reflected and refracted waves have been computed numerically for a specific model for different values of bonding parameter and results obtained graphically with angle of incidence of incident wave. Effects of bonding parameter, viscosity and stretch on the amplitude ratios are discussed.

Keywords: Amplitude ratio, bonding parameter, reflection, refraction, viscoelastic solid.

I. INTRODUCTION

The linear theory of micropolar elasticity developed by Nowacki [1] Micropolar elastic materials, roughly speaking, are the classical elastic materials with extra independent degree of freedom for the local rotations. These materials respond to spin inertia, body and surface couples and as a consequence they exhibit certain new static and dynamic effects, e.g. new types of waves and couples stresses. The micropolar theory of elasticity constructed by Eringen [2] and his co-workers intended to be applied on such materials and for problems where the ordinary theory of elasticity fails because of microstructure in the materials.

A micropolar elastic solid is distinguished from an elastic solid by the fact that it can support body and surface couples. These solids can undergo local deformations and micro-rotations such materials may be imagined as bodies which are made of rigid short cylinders or dumbbell type molecules.

From a continuum mechanical point of view, micropolar elastic solids may be characterized by a set of constitutive equations which define the elastic properties of such materials. A linear theory as a special case of the nonlinear theory of micro-elastic solids was first constructed by Eringen and Suhubi [3, 4].

Eringen [5] developed the theories of 'micropolar continua' and 'microstructures continua' which are special cases of the theory of 'micromorphic continua' earlier developed by Eringen and his co-workers [5]. Thus, the Eringen's '3M' theories (Micromorphic,

Microstretch, Micropolar) are the generalization the classical theory of elasticity. In classical continuum, each particle of a continuum is represented by a geometrical point and can have three degree of freedom of translation during the process of deformations.

Eringen's theory of micropolar elasticity keeps importance because of its applications in many physical substance for example material particles having rigid directors, chopped fibres composites, platelet composites,

ISBN: 978-93-86171-74-0

aluminium epoxy, liquid crystal with side chains, a large class of substance like liquid crystal with rigid molecules, rigid suspensions, animal blood with rigid cells, foams, porous materials, bones, magnetic fields, clouds with dust, concrete with sand and muddy fluids are example of micropolar materials.

The linear theory of micropolar viscoelasticity was developed by Eringen [6]. Mc Carthy & Eringen [7] discussed wave propagation conditions and growth equations. Kumar et al. [8] discussed a plane problem in a micropolar viscoelastic solid half space with stretch. Singh [9] discussed on reflection and transmission of plane harmonic waves at an interface between liquid and micropolar viscoelastic solid with stretch. Singh [10] have studied on reflection of plane at micropolar viscoelastic waves at a loosely bonded solid-solid interface. Kumari [11] discussed on propagation of elastic waves at a loosely interface of viscoelastic solid and fluid saturated porous solid. Recently, Gade and Raghukanth [12], Zhang et al. [13] and Merkel & Luding [14] discussed such waves and vibrations. The present paper is concerned with propagation of longitudinal wave at the solid solid interface, reflection and refraction of longitudinal waves at loosely bonded interface between micropolar elastic solid half space and micropolar viscoelastic solid half space with stretch.

II. BASIC EQUATIONS AND CONSTITUTIVE RELATIONS FOR MEDIUM M_1 (MICROPOLAR ELASTIC SOLID)

The equation of motion in micropolar elastic medium are given by Eringen [1] as

$$(c_1{}^2 + c_3{}^2)\nabla^2\phi = \frac{\partial^2\phi}{\partial t^2}, \tag{1}$$

$$(c_2{}^2 + c_3{}^2)\nabla^2 U + c_3{}^2\nabla \times \Phi = \frac{\partial^2 U}{\partial t^2}, \tag{2}$$

$$(c_4{}^2\nabla^2 - 2\omega_0{}^2)\Phi + \omega_0{}^2\nabla \times U = \frac{\partial^2\Phi}{\partial t^2}, \tag{3}$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \quad c_3^2 = \frac{\kappa}{\rho}, \quad c_4^2 = \frac{\gamma}{\rho j}, \quad \omega_0^2 = \frac{\kappa}{\rho j}, \tag{4}$$

Parfitt and Eringen [15] have shown that equation (1) corresponds to longitudinal wave propagating with velocity V_1, given by $V_1{}^2 = c_1{}^2 + c_3{}^2$ and equations. (2) - (3) are coupled equations in vector potentials U and Φ and these correspond to coupled transverse and micro-rotation waves. If $\frac{\omega^2}{\omega_0{}^2} > 20$, there exist two sets of coupled-wave propagating with velocities $1/\lambda_1$ and $1/\lambda_2$.

where

$$\lambda_1{}^2 = \frac{1}{2}\left[B - \sqrt{B^2 - 4C}\right], \quad \lambda_2{}^2 = \frac{1}{2}\left[B + \sqrt{B^2 - 4C}\right], \tag{5}$$

and

$$B = \frac{q(p-2)}{\omega^2} + \frac{1}{(c_2{}^2 + c_3{}^2)} + \frac{1}{c_4{}^2}, \qquad C = \left(\frac{1}{c_4{}^2} - \frac{2q}{\omega^2}\right)\frac{1}{(c_2{}^2 + c_3{}^2)},$$

$$p = \frac{\kappa}{\mu + \kappa}, \quad q = \frac{\kappa}{\gamma}. \tag{6}$$

 ISBN: 978-93-86171-74-0

We consider a two dimensional problem by taking the following components of displacement and micro-rotation as

$$U = (u, 0, w), \qquad \Phi = (0, \Phi_2, 0), \tag{7}$$

where

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \tag{8}$$

and components of stresses as

$$t_{zz} = (\lambda + 2\mu + \kappa)\frac{\partial^2 \phi}{\partial z^2} + \lambda \frac{\partial^2 \phi}{\partial x^2} + (2\mu + \kappa)\frac{\partial^2 \psi}{\partial x\, \partial z}, \tag{9}$$

$$t_{zx} = (2\mu + \kappa)\frac{\partial^2 \phi}{\partial x \partial z} - (\mu + \kappa)\frac{\partial^2 \psi}{\partial z^2} + \mu \frac{\partial^2 \psi}{\partial x^2} - \kappa \Phi_2, \tag{10}$$

$$m_{zy} = \gamma \frac{\partial \Phi_2}{\partial z}, \tag{11}$$

FOR MEDIUM M_2 (MICROPOLAR VISCOELASTIC SOLID WITH STRETCH)

Following Eringen [6, 16], the constitutive and field equations of micropolar viscoelastic solid with stretch, the absence of body forces and body couples, can be written as

$$\bar{t}_{kl} = \bar{\lambda}\bar{u}_{r,r}\delta_{kl} + \bar{\mu}(\bar{u}_{k,l} + \bar{u}_{l,k}) + \bar{\kappa}(\bar{u}_{l,k} - \epsilon_{klr}\bar{\phi}_r), \tag{12}$$

$$\bar{m}_{kl} = \bar{\beta}_0 \epsilon_{rkl}\bar{\Phi}_r + \bar{\alpha}\bar{\phi}_{r,r}\delta_{kl} + \bar{\beta}\bar{\phi}_{k,l} + \bar{\gamma}\bar{\phi}_{l,k}, \tag{13}$$

$$\bar{\beta}_k = \bar{\alpha}_0 \bar{\Phi}_{,k} + (\bar{\beta}_0/3)\epsilon_{rkl}\bar{\phi}_{r,l}, \tag{14}$$

and

$$(\bar{c}_1^2 + \bar{c}_3^2)\nabla(\nabla.\mathbf{u}) - (\bar{c}_2^2 + \bar{c}_3^2)\nabla \times (\nabla \times \mathbf{u}) + \bar{c}_3^2 \nabla \times \boldsymbol{\phi} = \ddot{\mathbf{u}} \tag{15}$$

$$(\bar{c}_4^2 + \bar{c}_5^2)\nabla(\nabla.\boldsymbol{\phi}) - \bar{c}_4^2 \nabla \times (\nabla \times \boldsymbol{\phi}) + \bar{\omega}_0^2 \nabla \times \mathbf{u} - 2\bar{\omega}_0^2 \boldsymbol{\phi} = \ddot{\boldsymbol{\phi}} \tag{16}$$

$$\bar{c}_6^2 \nabla^2 \bar{\Phi} - r_1 \bar{\Phi} = \ddot{\bar{\Phi}} \tag{17}$$

where

$$\bar{c}_1^2 = \frac{\bar{\lambda} + 2\bar{\mu}}{\bar{\rho}}, \qquad \bar{c}_2^2 = \frac{\bar{\mu}}{\bar{\rho}}, \qquad \bar{c}_3^2 = \frac{\bar{\kappa}}{\bar{\rho}}, \qquad \bar{c}_4^2 = \frac{\bar{\gamma}}{\bar{\rho}\,\bar{j}}, \qquad \bar{c}_5^2 = \frac{\bar{\alpha} + \bar{\beta}}{\bar{\rho}\,\bar{j}},$$

$$\bar{\omega}_0{}^2 = \frac{\bar{c}_3^2}{\bar{j}} = \frac{\bar{\kappa}}{\bar{\rho}\,\bar{j}}, \qquad \bar{c}_6^2 = \frac{2\bar{\alpha}_0}{\bar{\rho}\,\bar{j}}, \qquad r_1 = \frac{2\bar{\eta}_0}{\bar{\rho}\,\bar{j}}, \tag{18}$$

where symbols $\lambda, \mu, \gamma, \kappa, \rho, j, \bar{\rho}, \bar{j}, \bar{\lambda}, \bar{\mu}, \bar{\kappa}, \bar{\gamma}, \bar{\alpha}_0, \bar{\beta}_0, \bar{\eta}_0$ have their usual meaning. $\mathbf{u}$, $\boldsymbol{\phi}$ and $\bar{\Phi}$ are displacement vector, micro-rotation vector and micro-stretch respectively. δ_{kl} is the kronecker delta. Superposed dots on the right hand side of (15), (16) and (17) stand for second partial derivative with respect to time.

Taking $\mathbf{u} = (\bar{u}, 0, \bar{w})$ and $\boldsymbol{\phi} = (0, \bar{\phi}_2, 0)$ and introducing potentials $\bar{\phi} = (x, z, t)$ and $\bar{\psi} = (x, z, t)$ which are related to displacement components as

$$\bar{u} = \left(\frac{\partial \bar{\phi}}{\partial z}\right) + \left(\frac{\partial \bar{\psi}}{\partial x}\right), \qquad \bar{w} = \left(\frac{\partial \bar{\phi}}{\partial x}\right) - \left(\frac{\partial \bar{\psi}}{\partial z}\right), \tag{19}$$

and components of stresses as

 ISBN: 978-93-86171-74-0

$$\bar{t}_{zz} = -(\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\frac{\partial^2\bar{\psi}}{\partial z^2} + (2\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})\frac{\partial^2\bar{\phi}}{\partial x\,\partial z} + \bar{\lambda}\frac{\partial^2\bar{\psi}}{\partial x^2}, \tag{20}$$

$$\bar{t}_{zx} = (\bar{\lambda} + \bar{\kappa})\frac{\partial^2\bar{\phi}}{\partial z^2} + \bar{\mu}\frac{\partial^2\bar{\phi}}{\partial x^2} + \bar{\kappa}\frac{\partial^2\bar{\psi}}{\partial z\,\partial x} - \bar{\kappa}\bar{\phi}_2, \tag{21}$$

$$\bar{m}_{zy} = -\bar{\beta}_0\bar{\Phi}_{,x} + \bar{\beta}\bar{\phi}_{2,y} + \bar{\gamma}\bar{\phi}_{2,z}, \tag{22}$$

$$\bar{\beta}_z = \bar{\alpha}_0\bar{\Phi}_{,z} + (\bar{\beta}_0/3)\bar{\phi}_{2,x}, \tag{23}$$

Substituting the displacement components given by (19) in (15) to (17), we obtained

$$\left(\nabla^2 - \frac{1}{(\bar{c}_2{}^2 + \bar{c}_3{}^2)}\frac{\partial^2}{\partial t^2}\right)\bar{\phi} = 0, \tag{24}$$

$$\left(\nabla^2 - \frac{1}{(\bar{c}_2{}^2 + \bar{c}_3{}^2)}\frac{\partial^2}{\partial t^2}\right)\bar{\psi} - \bar{p}\bar{\phi}_2 = 0, \tag{25}$$

$$\left(\nabla^2 - 2\bar{q} - \frac{1}{\bar{c}_4{}^2}\frac{\partial^2}{\partial t^2}\right)\bar{\phi}_2 + \bar{q}\nabla^2\bar{\psi} = 0, \tag{26}$$

$$\left(\nabla^2 - 2r_1 - \frac{1}{\bar{c}_6{}^2}\frac{\partial^2}{\partial t^2}\right)\bar{\Phi} = 0, \tag{27}$$

where

$$\bar{p} = \frac{\bar{\mu}}{(\bar{\mu} + \bar{\kappa})}, \qquad \bar{q} = \frac{\bar{\kappa}}{\bar{\gamma}}, \tag{28}$$

Assume harmonic time variation as

$$\bar{\phi}(x,z,t) = \bar{\phi}(x,z)\exp(i\bar{\omega}t),$$
$$\bar{\psi}(x,z,t) = \bar{\psi}(x,z)\exp(i\bar{\omega}t),$$
$$\bar{\phi}_2(x,z,t) = \bar{\phi}_2(x,z)\exp(i\bar{\omega}t),$$
$$\bar{\Phi}(x,z,t) = \bar{\Phi}(x,z)\exp(i\bar{\omega}t), \tag{29}$$

Substituting (29) in (24) to (27), we get

$$\left(\nabla^2 + \frac{\bar{\omega}^2}{\bar{V}_1^2}\right)\bar{\phi} = 0, \tag{30}$$

$$(\nabla^4 + \bar{\omega}^2\bar{B}\nabla^2 + \bar{\omega}^4\bar{C})(\bar{\psi}, \bar{\phi}_2) = 0, \tag{31}$$

$$\left(\nabla^2 + \frac{\bar{\omega}^2}{V^2}\right)\bar{\Phi} = 0, \tag{32}$$

where

$$\bar{B} = \frac{\bar{q}(\bar{p} - 2)}{\bar{\omega}^2} + \frac{1}{(\bar{c}_2^2 + \bar{c}_3^2)} + \frac{1}{\bar{c}_4^2}, \qquad \frac{1}{(\bar{c}_2^2 + \bar{c}_3^2)}\left(\frac{1}{\bar{c}_4^2} - \frac{2\bar{q}}{\bar{\omega}^2}\right), \tag{33}$$

and

$$V^2 = \frac{\bar{c}_6^2}{\left(1 - \frac{r_1\bar{c}_6^2}{\bar{\omega}^2}\right)}, \qquad \bar{V}_1^2 = \bar{c}_1^2 + \bar{c}_3^2, \tag{34}$$

 ISBN: 978-93-86171-74-0

In an unbounded medium, the solution of (30) corresponds to modified longitudinal displacement wave propagating with velocity V_1.

The solution of (31) can be written as

$$\bar{\psi} = \bar{\psi}_1 + \bar{\psi}_2 \tag{35}$$

where $\bar{\psi}_1$ and $\bar{\psi}_2$ satisfy

$$\left(\nabla^2 + \bar{\delta}_1^2\right)\bar{\psi}_1 = 0, \tag{36}$$

$$\left(\nabla^2 + \bar{\delta}_2^2\right)\bar{\psi}_2 = 0, \tag{37}$$

and

$$\bar{\delta}_1^2 = \bar{\lambda}_1^2 \bar{\varpi}^2, \qquad \bar{\delta}_2^2 = \bar{\lambda}_2^2 \bar{\varpi}^2, \tag{38}$$

$$\bar{\lambda}_{1,2}^2 = \frac{\left[\bar{B} \pm \sqrt{(\bar{B}^2 - 4\bar{C})}\right]}{2}. \tag{39}$$

From (31) we obtain

$$\bar{\phi}_2 = \bar{E}\bar{\psi}_1 + \bar{F}\bar{\psi}_2,$$

where

$$\bar{E} = \frac{\left(\frac{\bar{\varpi}^2}{(\bar{c}_2^2 + \bar{c}_3^2)} - \bar{\delta}_1^2\right)}{\bar{p}}, \qquad \bar{F} = \frac{\left(\frac{\bar{\varpi}^2}{(\bar{c}_2^2 + \bar{c}_3^2)} - \bar{\delta}_2^2\right)}{\bar{p}}. \tag{40}$$

Thus there are two waves propagating with velocities $\bar{\lambda}_1^{-1}$ and $\bar{\lambda}_2^{-1}$ each consisting of transverse displacement $\bar{\psi}$ and transverse micro-rotation$\bar{\phi}_2$. Following Parfitt & Eringen [15], we call these waves the modified coupled transverse displacement wave and the transverse micro-rotational wave, respectively.

Equation (32) shows that there exists a wave propagating with velocity V, which we call a longitudinal micro-stretch wave in a micropolar viscoelastic medium with stretch.

This velocity V is real and finite if

$$1 - \left(\frac{r_1 \bar{c}_6^2}{\bar{\varpi}^2}\right) > 0. \tag{41}$$

The inequality (41) with help of (18) reduces to

$$\bar{\varpi} > \bar{\varpi}_c, \tag{42}$$

where

$$\bar{\varpi}_c = \sqrt{2}\ \frac{\bar{\eta}_0}{\bar{\rho}\,\bar{j}}$$

This is the condition for the existence of a modified longitudinal micro-stretch wave (LMS-wave).

III. FORMULATION OF THE PROBLEM

Consider a two dimensional problem by taking the z-axis pointing into the lower half-space and the plane interface z=0 separating the uniform micropolar elastic solid half space M_1 [z>0] and the micropolar

ISBN: 978-93-86171-74-0

viscoelastic solid half space with stretch M_2 [z<0]. A longitudinal wave propagates through the medium M_1 and incident at the plane z=0 and making an angle θ_0 with normal to the surface. Corresponding to incident longitudinal wave, we get three reflected waves in the medium M_1 and four refracted waves in medium M_2. See fig.1

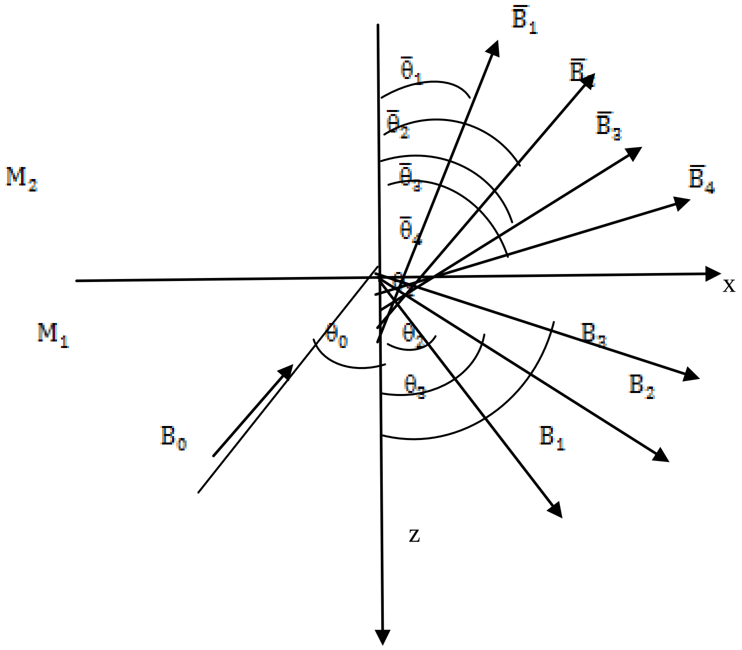


Fig.1Geometry of the problem.

IN MEDIUM M_1

$$\phi = B_0 \exp\{ik_0 (x \sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} + B_1 \exp\{ik_0 (x \sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}, \tag{43}$$

$$\psi = B_2 \exp\{i\delta_1 (x \sin\theta_2 + z\cos\theta_2) + i\omega_2 t\} + B_3 \exp\{i\delta_2 (x \sin\theta_3 + z\cos\theta_3) + i\omega_3 t\}, \tag{44}$$

$$\Phi_2 = EB_2 \exp\{i\delta_1 (x \sin\theta_2 + z\cos\theta_2) + i\omega_2 t\} + FB_3 \exp\{i\delta_2 (x \sin\theta_3 + z\cos\theta_3) + i\omega_3 t\}, \tag{45}$$

where

$$E = \frac{\delta_1^2\left(\delta_1^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}}, \tag{46}$$

$$F = \frac{\delta_2^2\left(\delta_2^2 - \frac{\omega^2}{(c_2{}^2 + c_3{}^2)} + pq\right)}{\text{deno.}}, \tag{47}$$

and

$$\text{deno.} = p\left(2q - \frac{\omega^2}{c_4{}^2}\right), \quad \delta_1^2 = \lambda_1^2\omega^2, \quad \delta_2^2 = \lambda_2^2\omega^2\,. \tag{48}$$

 ISBN: 978-93-86171-74-0

where B_0, B_1, B_2, B_3 are amplitudes of incident longitudinal wave, reflected longitudinal wave, reflected coupled transverse and reflected micro-rotation waves respectively.

IN MEDIUM M_2

$$\bar{\phi} = \bar{B}_1 \exp\{i\bar{k}_0 (x \sin\bar{\theta}_1 - z \cos\bar{\theta}_1) + i\bar{\omega}_1 t\}, \tag{49}$$

$$\bar{\psi} = \bar{B}_2 \exp\{i\bar{\delta}_1 (x \sin\bar{\theta}_2 - z \cos\bar{\theta}_2) + i\bar{\omega}_2 t\} + \bar{B}_3 \exp\{i\bar{\delta}_2 (x \sin\bar{\theta}_3 - z \cos\bar{\theta}_3) + i\bar{\omega}_3 t\}, \tag{50}$$

$$\bar{\phi}_2 = \bar{E}\,\bar{B}_2 \exp\{i\bar{\delta}_1 (x \sin\bar{\theta}_2 - z \cos\bar{\theta}_2) + i\bar{\omega}_2 t\} + \bar{F}\bar{B}_3 \exp\{i\bar{\delta}_2 (x \sin\bar{\theta}_3 - z \cos\bar{\theta}_3) + i\bar{\omega}_3 t\}, \tag{51}$$

$$\bar{\Phi} = G\bar{B}_0 \exp\{i\bar{k}_4 (x \sin\bar{\theta}_4 - z \cos\bar{\theta}_4) + i\bar{\omega}_4 t\}, \tag{52}$$

where $\bar{B}_1, \bar{B}_2, \bar{B}_3, \bar{B}_4$ are amplitudes of refracted longitudinal displacement wave, two refracted sets of two coupled waves (CD I and CD II) and refracted longitudinal micro-stretch wave (LMS-wave) respectively. G is the constant of dimension L^{-2}.

IV. BOUNDARY CONDITIONS

At the interface between micropolar elastic solid and micropolar viscoelastic solid with stretch, the appropriate boundary conditions are continuity of force stresses, couple stresses, displacements and microrotation, and vector first moment respectively. Mathematically, these boundary conditions at the interface z = 0, can be written as:

$$t_{zz} = \bar{t}_{zz}, \quad t_{zx} = \bar{t}_{zx}, \quad m_{zy} = \bar{m}_{zy}, \quad \bar{t}_{zx} = K_t(u - \bar{u}),$$

$$w = \bar{w}, \quad \Phi_2 = \bar{\phi}_2, \quad \bar{\beta}_z = 0. \tag{53}$$

where $K_t = i\bar{\kappa}\bar{\mu}\tau$ and $\tau = \xi/(1-\xi)\sin\theta_0$, ξ is bonding constant.

$0 \le \xi \le 1$. $\xi = 0$ corresponds to smooth surface and $\xi = 1$ corresponds to a welded interface.

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{\lambda_1^{-1}} = \frac{\sin\theta_3}{\lambda_2^{-1}} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{\lambda}_1^{-1}}, = \frac{\sin\bar{\theta}_3}{\bar{\lambda}_2^{-1}} = \frac{\sin\bar{\theta}_4}{V}, \tag{54}$$

For longitudinal wave,

$$V_0 = V_1, \theta_0 = \theta_1, \tag{55}$$

Also

$$k_0 V_1 = \delta_1 \lambda_1^{-1} = \delta_2 \lambda_2^{-1} = \bar{k}_0 \bar{V}_1 = \bar{\delta}_1 \bar{\lambda}_1^{-1} = \bar{\delta}_2 \bar{\lambda}_2^{-1} = \bar{k}_4 V = \omega, \quad \text{at } z = 0 \tag{56}$$

Making the use of potentials given by equations (43)-(45) and (49)-(52) in the boundary conditions given by (53) and using (54)-(56), we get a system of seven non homogeneous equations which can be written as:

$$\sum_{j=1}^{7} a_{ij} Z_j = Y_i, \quad (i = 1,2,3,4,5,6,7) \tag{57}$$

where

 ISBN: 978-93-86171-74-0

$$Z_1 = \frac{B_1}{B_0}, Z_2 = \frac{B_2}{B_0}, Z_3 = \frac{B_3}{B_0}, Z_4 = \frac{\bar{B}_1}{B_0}, Z_5 = \frac{\bar{B}_2}{B_0}, Z_6 = \frac{\bar{B}_3}{B_0}, Z_7 = \frac{\bar{B}_4}{B_0}, \qquad (58)$$

where Z_1 to Z_7 are the amplitude ratios of reflected longitudinal wave, reflected coupled-wave at an angle θ_2, reflected coupled-wave at an angle θ_3, refracted longitudinal displacement wave, two refracted sets of two coupled waves (CD I and CD II) and refracted LMS-waves respectively. Also a_{ij} and Y_i in non-dimensional form are as

$$a_{11} = \frac{\lambda}{\mu} + D_2\cos^2\theta_1, \quad a_{12} = D_2\sin\theta_2\cos\theta_2\frac{\delta_1^2}{k_0^2}, \quad a_{13} = D_2\sin\theta_3\cos\theta_3\frac{\delta_2^2}{k_0^2},$$

$$a_{14} = \left\{\frac{(2\bar{\lambda} + 2\bar{\mu} + \bar{\kappa})}{\mu}\sin\bar{\theta}_1\cos\bar{\theta}_1\right\}\frac{\bar{k}_0^2}{k_0^2}, \quad a_{15} = \frac{(2\bar{\mu} + \bar{\kappa})}{\mu}\left(\cos^2\bar{\theta}_2 - \sin^2\bar{\theta}_2\right)\frac{\bar{\delta}_1^2}{k_0^2},$$

$$a_{16} = \frac{(2\bar{\mu} + \bar{\kappa})}{\mu}\left(\cos^2\bar{\theta}_3 - \sin^2\bar{\theta}_3\right)\frac{\bar{\delta}_2{}^2}{k_0^2}, \quad a_{17} = 0, \quad Y_1 = -a_{11}.$$

$$a_{21} = D_2\sin\theta_1\cos\theta_1, \quad a_{22} = -\left\{(D_1\cos^2\theta_2 - \sin^2\theta_2) - \frac{\kappa}{\mu}\frac{E}{\delta_1^2}\right\}\frac{\delta_1^2}{k_0^2},$$

$$a_{23} = -\left\{(D_1\cos^2\theta_3 - \sin^2\theta_3) - \frac{\kappa}{\mu}\frac{F}{\delta_2^2}\right\}\frac{\delta_2^2}{k_0^2}, \quad a_{24} = -\left\{\frac{(\bar{\mu} + \bar{\kappa}\cos^2\bar{\theta}_1)}{\mu}\right\}\frac{\bar{k}_0^2}{k_0^2},$$

$$a_{25} = \left(\bar{\delta}_1^2\sin\bar{\theta}_2\cos\bar{\theta}_2 - \bar{E}\right)\frac{\bar{\kappa}}{\mu k_0^2}, \quad a_{26} = \left(\bar{\delta}_2^2\sin\bar{\theta}_3\cos\bar{\theta}_3 - \bar{F}\right)\frac{\bar{\kappa}}{\mu k_0^2},$$

$$a_{27} = 0, \ Y_2 = a_{21}.$$

$$a_{31} = 0, \quad a_{32} = E\gamma\cos\theta_2\frac{\delta_1}{k_0}, \quad a_{33} = F\gamma\cos\theta_3\frac{\delta_2}{k_0}, \quad a_{34} = 0,$$

$$a_{35} = \bar{E}\bar{\gamma}\cos\bar{\theta}_2\frac{\bar{\delta}_1}{k_0}, \quad a_{36} = \bar{F}\bar{\gamma}\cos\bar{\theta}_3\frac{\bar{\delta}_2}{k_0}, \quad a_{37} = G\bar{\beta}_0\sin\bar{\theta}_4\frac{\bar{k}_4}{k_0}, \quad Y_3 = a_{31}.$$

$$a_{41} = -K_t\sin\theta_1, \quad a_{42} = K_t\cos\theta_2\frac{\delta_1}{k_0}, \quad a_{43} = K_t\cos\theta_3\frac{\delta_2}{k_0},$$

$$a_{44} = \left\{K_t\cos\bar{\theta}_1\frac{\bar{k}_0}{k_0} + (\bar{\mu} + \bar{\kappa}\cos^2\bar{\theta}_1)\frac{\bar{k}_0^2}{i\,k_0}\right\},$$

$$a_{45} = -\left\{K_t\sin\bar{\theta}_2\frac{\bar{\delta}_1}{k_0} + \frac{\bar{\kappa}}{k_0 i}\left(\bar{\delta}_1^2\sin\bar{\theta}_2\cos\bar{\theta}_2 - \bar{E}\right)\right\},$$

$$a_{46} = -\left\{K_t\sin\bar{\theta}_3\frac{\bar{\delta}_1}{k_0} + \frac{\bar{\kappa}}{k_0 i}\left(\bar{\delta}_2^2\sin\bar{\theta}_3\cos\bar{\theta}_3 - \bar{F}\right)\right\}, \quad a_{47} = 0, \quad Y_4 = -a_{41}.$$

$$a_{51} = \cos\theta_1, \quad a_{52} = \sin\theta_2\frac{\delta_1}{k_0}, \quad a_{53} = \sin\theta_3\frac{\delta_2}{k_0}, \quad a_{54} = -\sin\bar{\theta}_1\frac{\bar{k}_0}{k_0},$$

$$a_{55} = -\cos\bar{\theta}_2\frac{\bar{\delta}_1}{k_0}, \quad a_{46} = -\cos\bar{\theta}_3\frac{\bar{\delta}_2}{k_0}, \quad a_{57} = 0, \quad Y_5 = a_{51}.$$

$$a_{61} = 0, \quad a_{62} = -E, \quad a_{63} = -F, \quad a_{64} = 0, \quad a_{65} = \bar{E}, \quad a_{66} = \bar{F},$$

$$a_{67} = 0, \quad Y_6 = a_{61}.$$

$$a_{71} = a_{72} = a_{73} = a_{74} = 0, \quad a_{75} = \bar{E}\sin\bar{\theta}_2 \frac{\bar{\beta}_0}{3}\frac{\bar{\delta}_1}{k_0}, \quad a_{76} = \bar{F}\sin\bar{\theta}_3 \frac{\bar{\beta}_0}{3}\frac{\bar{\delta}_2}{k_0},$$

$$a_{77} = -\bar{\alpha}_0 G\cos\bar{\theta}_4 \frac{\bar{k}_4}{k_0}, \quad Y_7 = a_{71}. \tag{59}$$

where

$$D_1 = 1 + \frac{\lambda}{\mu}, \; D_2 = 1 + D_1.$$

V. NUMERICAL RESULTS AND DISCUSSION

The theoretical results obtained above indicate that the amplitude ratios $Z_i (i = 1,2,3,4,5,6,7)$ depend on the angle of incidence of incident wave and elastic properties of half spaces. In order to study in more detail the behaviour of various amplitude ratios, we take the case of aluminium-epoxy composite subject to viscous effect and stretch effect for our calculation. Following Gauthier [17], the physical constants for micropolar elastic solid are

$$\lambda = 7.59 \times 10^{11} \text{dyne/cm}^2, \quad \mu = 1.89 \times 10^{11} \text{dyne/cm}^2,$$

$$\kappa = 0.0149 \times 10^{11} \text{dyne/cm}^2, \quad \rho = 2.19 \text{gm/cm}^3$$

$$\gamma = 0.0268 \times 10^{11} \text{ dyne}, \quad j = 0.0196 \text{ cm}^2, \quad \frac{\omega^2}{\omega_0{}^2} = 20. \tag{60}$$

For a particular modal micropolar viscoelastic solid with stretch, the physical constants are given as

$$\lambda^* = 6.8 \times 10^{11} \text{dyne/cm}^2, \quad \mu^* = 1.63 \times 10^{11} \text{dyne/cm}^2,$$

$$\kappa^* = 0.0134 \times 10^{11} \text{dyne/cm}^2, \quad \bar{\rho} = 2.06 \text{gm/cm}^3,$$

$$\gamma^* = 0.0268 \times 10^{11} \text{ dyne}, \quad \bar{j} = 0.0196 \text{ cm}^2, \quad \frac{\bar{\omega}^2}{\bar{\omega}_0{}^2} = 20 \tag{61}$$

$$\bar{\lambda} = \lambda^*(1 + iQ_1^{-1}), \quad \bar{\mu} = \mu^*(1 + iQ_2^{-1}),$$

$$\bar{\kappa} = \kappa^*(1 + iQ_3^{-1}), \quad \bar{\gamma} = \gamma^*(1 + iQ_4^{-1}).$$

where Q_i (i=1, 2, 3, 4) are chosen arbitrary as

$$Q_1 = 4, \quad Q_2 = 9, \quad Q_3 = 13, \quad Q_4 = 11.$$

and

$$\alpha_0 = 9.15 \times 10^5 \text{dyne}, \quad \beta_0 = 7.26 \times 10^5 \text{dyne}, \quad \eta_0 = 5.32 \times 10^5 \text{dyne}.$$

A computer programme in MATLAB has been developed to calculate the modulus of amplitude ratios of various reflected and refracted waves for the particular model and to depict graphically. $Z_i (i = 1,2,3.)$ and $Z_i (i = 4,5,6,7.)$ represents the modulus of amplitude ratios for reflected and refracted waves respectively. Dashed dotted line represents the variations of the amplitude ratios for the bonding constant $\xi = 0$, dashed line for $\xi = 0.5$ and bold dotted line for $\xi = 1$ in all the figures (2) – (22) with respect to angle of incidence of the incident longitudinal wave. The variations in all the figures are shown for the range $0^\circ \le \theta \le 90^\circ$.

Figures (2) – (22) represent the variations of the amplitudes ratios of reflected and refracted waves with an angle of incidence of incident wave. In such circumstances, there are three cases arise (micropolar viscoelastic solid

 ISBN: 978-93-86171-74-0

with stretch lying over micropolar elastic solid half-space, micropolar elastic solid with stretch lying over micropolar elastic solid half-space, micropolar viscoelastic solid without stretch lying over micropolar elastic solid half-space).

CASE I. (MICROPOLAR VISCOELASTIC SOLID WITH STRETCH LYING OVER MICROPOLAR ELASTIC SOLID)

Figure (2) depicts the variations of amplitude ratios for reflected longitudinal displacement wave LD wave with the angle of incidence of incident wave for ξ = 0.0, 0.5 and 1.0. For each bonding parameter, the amplitude ratios reflected LD wave first decrease to their minimum values and then attain their respective maxima. The amplitude ratios for reflected LD wave vary with the change in the value of bonding parameter at each angle of incidence.

Figure (3) shows the variations of amplitude ratios for reflected coupled wave CD I wave with the angle of incidence of incident wave for ξ = 0.0, 0.5 and 1.0. For each bonding parameter, the amplitude ratios reflected CD I wave first increase to their maximum values and then attain their respective minima. The effect of bonding parameter is clear from the figure.

Figure (4) shows the variations of amplitude ratios for reflected coupled wave CD II wave with the angle of incidence of incident wave. For ξ = 0.0, behaviour is oscillating and for the parameters 0.5 and 1.0, the amplitude ratio is almost constant.

Figure (5) shows the variations of amplitude ratios for refracted LD wave with the angle of incidence of incident wave. For ξ = 0.5 and 1.0, the amplitude ratio first attain its maximum value then decrease and attain its minimum value. For the parameter ξ =0.0, the amplitude ratio for refracted LD wave is constant.

Figures (6) – (8) shows the variations of amplitude ratios for two refracted coupled waves (CD I - CD II) and refracted LMS waves respectively, with the angle of incidence of incident wave, for ξ = 0.0, 0.5 and 1.0. For each bonding parameter, the amplitude ratios for refracted CD I, refracted CD II and refracted LMS waves respectively, first increase to their maximum values and then attain their respective minima.

CASE II. (MICROPOLAR ELASTIC SOLID WITH STRETCH LYING OVER MICROPOLAR ELASTIC SOLID)

Comparing the figures (2) – (3) to (9) – (11), the effect of viscosity is clear. In figure (9) depicts the variations of amplitude ratios for reflected LD wave with the angle of incidence of incident wave for ξ = 0.0 and 0.5. For bonding parameters, the amplitude ratios reflected LD wave first decrease to their minimum values and then attain their respective maxima.

Figure (10) shows the variations of amplitude ratios for reflected coupled wave CD I wave with the angle of incidence of incident wave. For bonding parameter ξ = 0.0, the amplitude ratio reflected CD I wave first increase to their maximum values and then attain their respective minima but for ξ = 0.5, the behaviour is oscillatory.

 ISBN: 978-93-86171-74-0

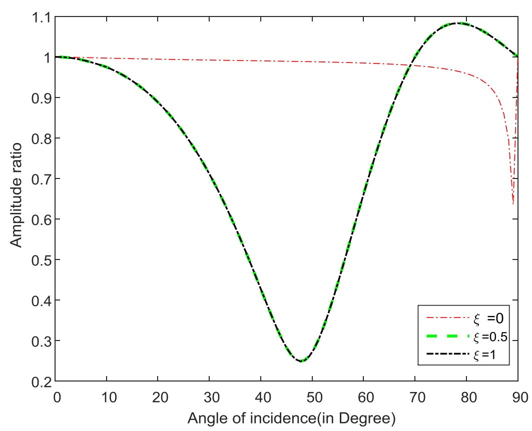
Figure (11) shows the variations of amplitude ratio for reflected coupled wave CD II wave with the angle of incidence of incident wave. For ξ = 0.0, smoothly decrease and attain their minimum value and for the parameter 0.5, the amplitude ratio reflected CD II the behaviour is oscillatory.

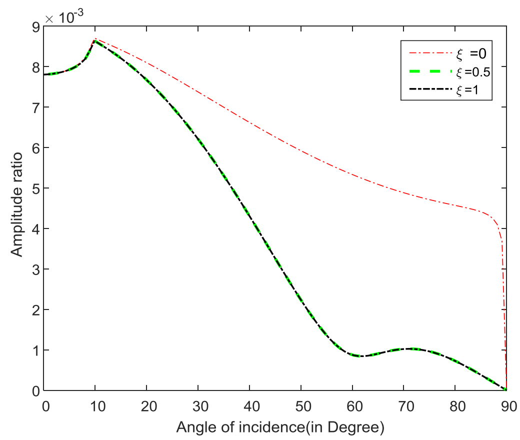
For bonding parameter ξ = 0.5, the amplitude ratios for refracted LD, refracted CD I, reflected CD II and refracted LMS-waves are shown in the figures (12) – (15) and also clearly shows the effect of viscous. Figure (12) shows the variations of amplitude ratios for refracted LD wave with the angle of incidence of incident wave. For ξ = 0.0, the amplitude ratio is almost constant and for the parameter ξ =0.5, the amplitude ratios refracted LD wave first increase to their maximum values at angle 58° and then attain their respective minima.

Figures (13) – (15) shows the variations of amplitude ratios for refracted CD I, refracted CD II and refracted LMS waves respectively, with the angle of incidence of incident wave. For bonding parameter ξ = 0.0, the amplitude ratio for refracted CD I, refracted CD II and refracted LMS waves respectively, first increase to their maximum values and then attain their respective minima and for ξ = 0.5, the behaviour is oscillatory.

CASE III. (MICROPOLAR VISCOELASTIC SOLID WITHOUT STRETCH LYING OVER MICROPOLAR ELASTIC SOLID)

Comparing the figures (2) – (8) to corresponding figures (16) – (22) the effect of stretch of medium II is negligible on amplitude ratios.





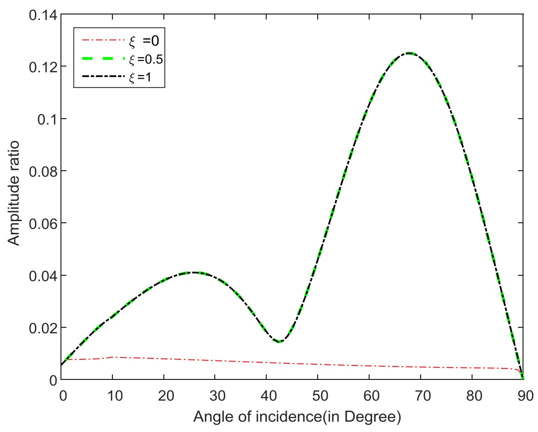


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Fig.2-4.Variation of the amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident longitudinal wave.

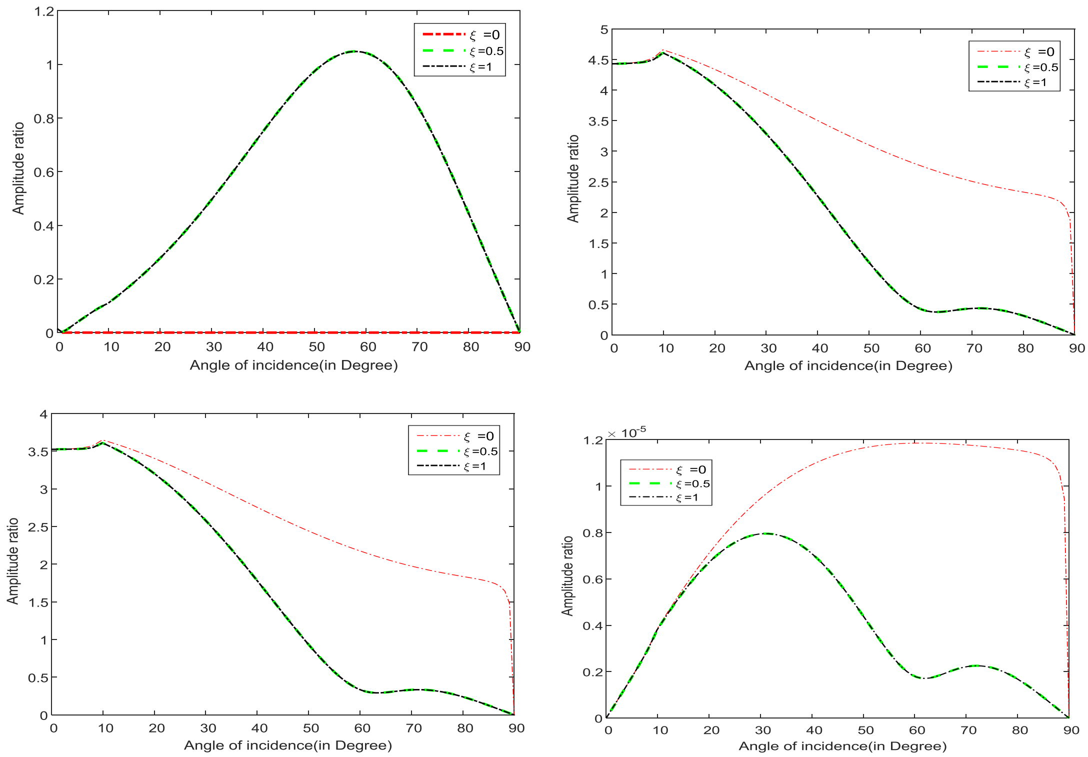

Fig.5-8.Variation of the amplitudes ratios $|Z_i|$, $(i = 4, 5, 6, 7)$ with angle of incidence of the incident longitudinal wave.

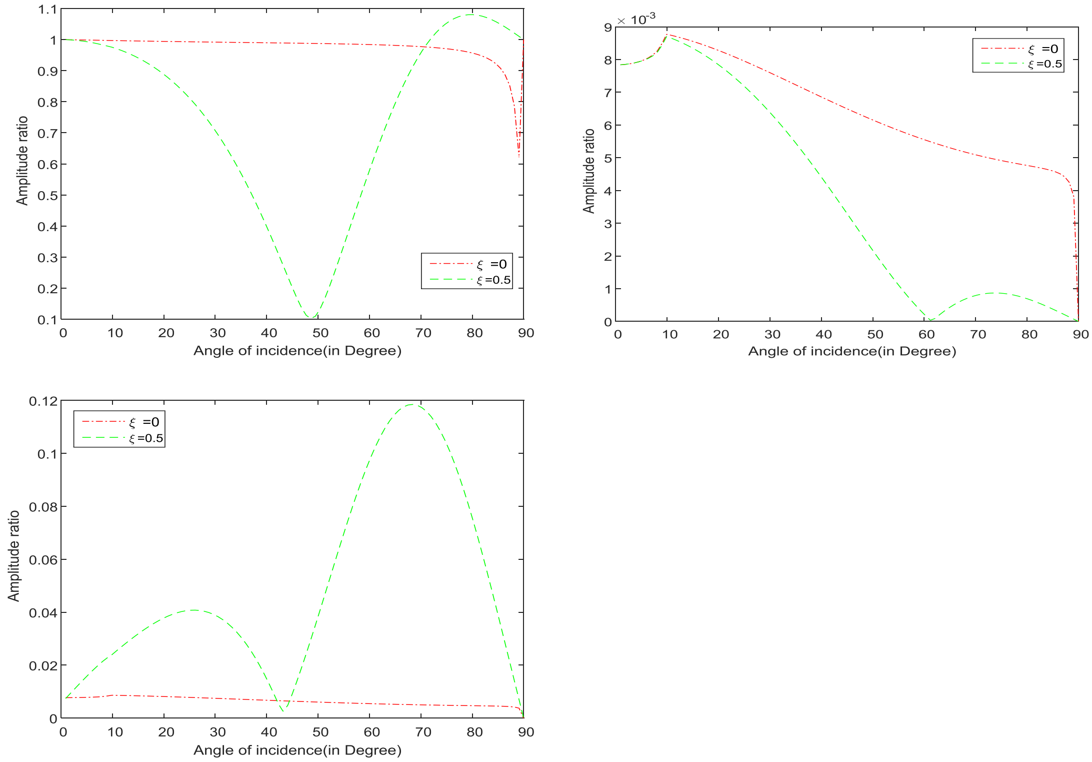

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Fig.9-11.Variation of the amplitudes ratios $|Z_i|$, $(i = 1, 2, 3)$ with angle of incidence of the incident longitudinal wave.

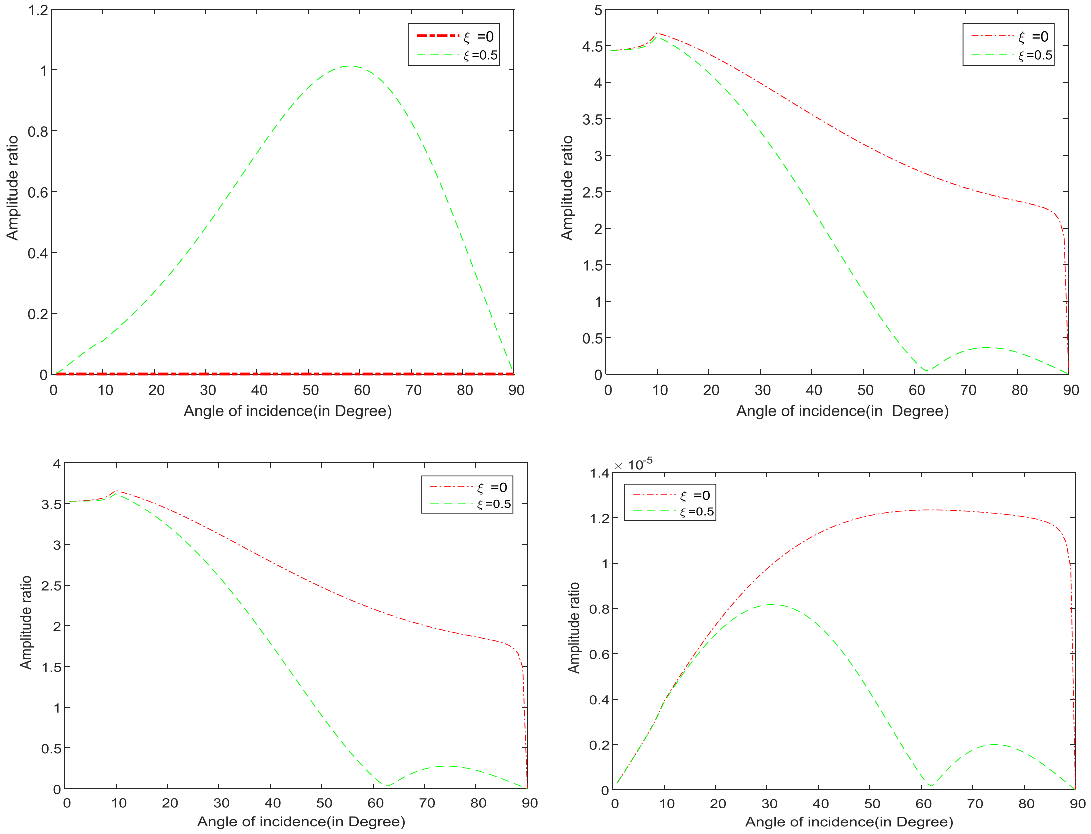
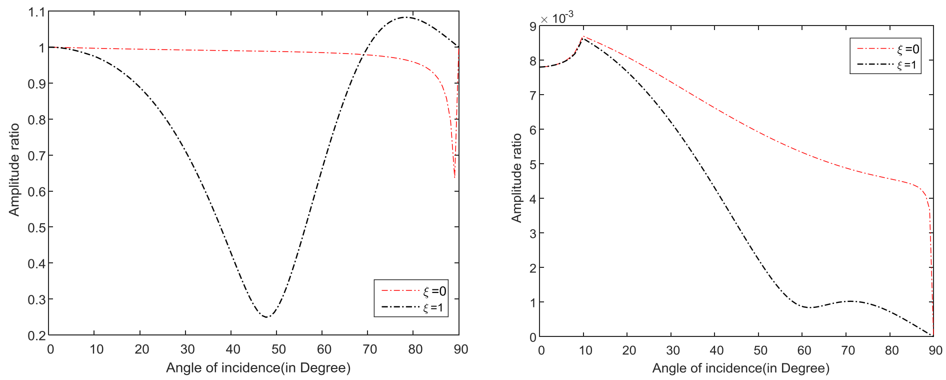


Fig.12-15.Variation of the amplitudes ratios $|Z_i|$, $(i = 4, 5, 6, 7)$ with angle of incidence of the incident longitudinal wave.



 ISBN: 978-93-86171-74-0

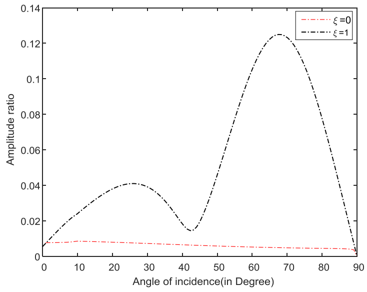


Fig.16-18.Variation of the amplitudes ratios $|Z_i|$, $(i = 1, 2, 3.)$ with angle of incidence of the incident longitudinal wave.

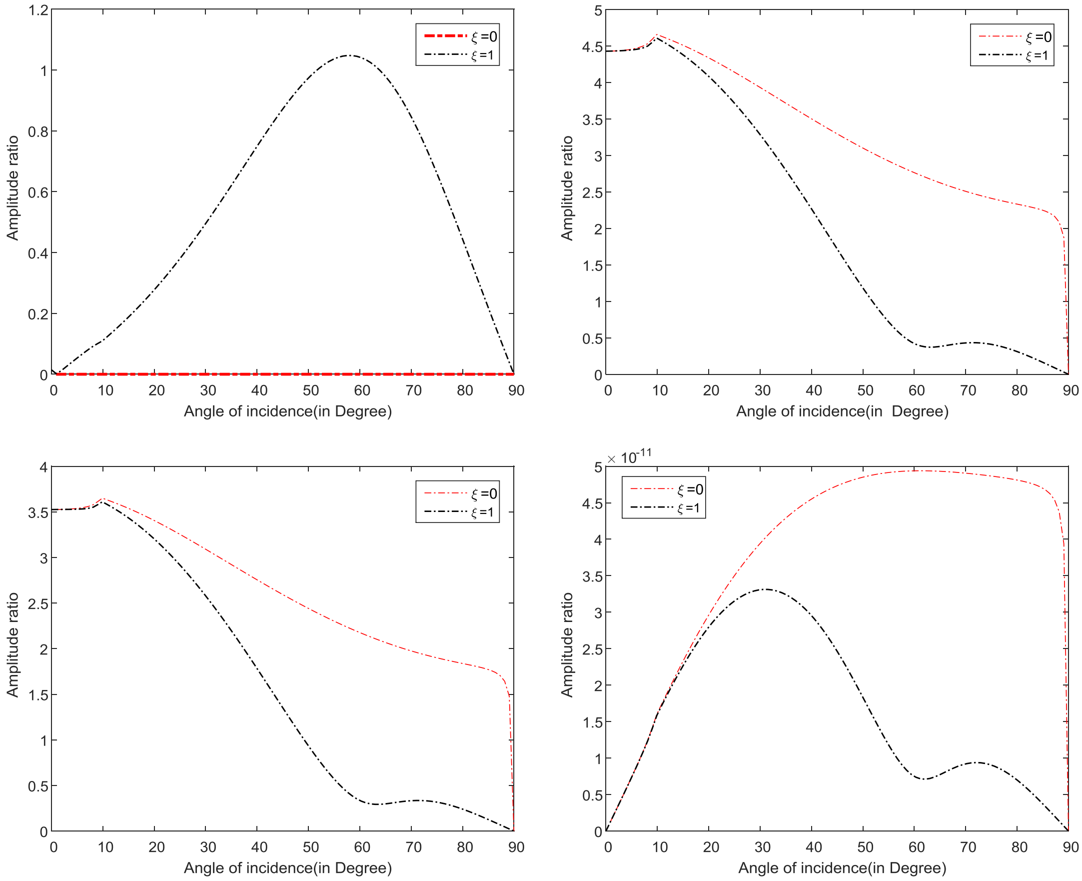


Fig.19-22.Variation of the amplitudes ratios $|Z_i|$, $(i = 1, 2, 3.)$ with angle of incidence of the incident longitudinal wave.

VI. CONCLUSION

In conclusion, a mathematical study of reflection and refraction coefficients at loosely bonded interface separating micropolar elastic solid half space and micropolar viscoelastic solid with stretch half space is made when longitudinal wave is incident. It is observed that

1. The amplitudes ratios of various reflected and refracted waves are found to be complex valued.

2. The modulus of amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties of half spaces.
3. The effects of viscous and stretch are significant on the amplitudes ratios.
4. The effect of incident wave is significant on amplitude ratios. All the amplitudes ratios are found to depend on incident waves.
5. The effect of bonding parameter for loosely bonded interface is significant when longitudinal wave is incident.
6. The solution leads to the existence of a new wave which named as longitudinal micro-stretch wave (LMS).

The model presented in this paper is one of the further realistic forms of the earth models. The present theoretical results may provide useful information for investigational scientists, researchers and seismologists are working in the area of wave propagation in micropolar elastic solid and micropolar viscoelastic solid with stretch.

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