STUDY OF LAMINATED COMPOSITE BEAM BY USING CLASSICAL LAMINATE THEORY

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ABSTRACT

Laminated composite beams and plates are commonly used in automotive, naval, aircraft, lightweight structures, aerospace exploration and civil engineering applications. Composite materials have interesting properties such as high strength to weight ratio, ease of fabrication, good electrical and thermal properties compared to metals. A laminated composite material consists of several layers of a composite mixture consisting of matrix and fibers. Each layer may have similar or dissimilar material properties with different fibre orientations under varying stacking sequence. There are many open issues relating to design of these laminated composites. Design engineer must consider several alternatives such as best stacking sequence, optimum fibre angles in each layer as well as number of layers itself based on criteria such as achieving highest natural frequency or largest buckling loads of such structure. Analysis of such composite materials starts with estimation of resultant material properties. Both classical theory and numerical methods such as finite element modelling may be employed in this line. Further, these estimated properties are to be used for computing the dynamic properties of the members made-up of these materials as equivalent isotropic members.. The obtained constants are validated with an ANSYS model, where the laminate stacking sequence is built and the member is subjected to a uniform strain at free end, while the reaction stress at the fixed end is predicted. The developed interface simplifies the design process to some extent. The dynamic analysis in terms of fundamental natural frequency and critical buckling load is illustrated by using these overall material constants as a later part of analysis.

I.INTRODUCTION

Classification of Composites

Composite materials in general are categorised based on the kind of reinforcements or the surrounding matrix. There are four commonly accepted types of composite materials based on reinforcements-

a) Fibrous composite materials that consist of fibres in a matrix.
b) Laminated composite materials that consist of layers of various materials.
c) Particulate composite materials that are composed of particles in a matrix.
d) Combinations of some or all of the first three types.

And the major composite classes based on structural composition of the matrix are

a) Polymer-Matrix Composites
b) Metal- Matrix Composites
c) Ceramic- Matrix Composites

d) Carbon- Carbon Composites
e) Hybrid Composites

Basic Concepts of Composite Materials:-

Fibres: -Fibres are the principal constituent in a fibre-reinforced composite material. They occupy the largest volume fraction in a composite laminate and share the major portion of the load acting on a composite structure. Proper selection of the type, amount and orientation of fibres is very important, because it influences the following characteristics of a composite laminate.

a) Specific gravity
b) Tensile strength and modulus
c) Compressive strength and modulus
d) Fatigue strength and fatigue failure mechanisms
e) Electric and thermal conductivities
f) Cost

The various types of fibres currently in use are

a) Glass Fibres
b) Carbon Fibres
c) Aramid Fibres
d) Boron Fibres
e) Silicon Carbide Fibres

1.1.1 Matrix

In a composite material the fibres are surrounded by a thin layer of matrix material that holds the fibres permanently in the desired orientation and distributes an applied load among all the fibres. The matrix also plays a strong role in determining the environmental stability of the composite article as well as mechanical factors such as toughness and shear strength.The matrix binds the fibres together, holding them aligned in the important stressed directions. The matrix must also isolate the fibres from each other so that they can act as separate entities. The matrix should protect the reinforcing filaments from mechanical damage (e.g. abrasion) and from environmental attack. A ductilematrix will provide a means of slowing down or stopping cracks that might have originated at broken fibres; conversely, a brittle matrix may depend upon the fibres to act as matrix crack stoppers. Through the quality of its "grip" on the fibres (the interfacial bond strength), the matrix can also be an important means of increasing the toughness of the composite. Because the reinforcing fibres can be oriented during fabrication of item, composites can be tailored to meet increased load demands in specific directions.

1.1.5 Advantages of Composites over the Conventional Materials

1. High strength to weight ratio
2. High stiffness to weight ratio
3. High impact resistance
4. Better fatigue resistance

1.1.6 Limitations of Composites

1. Mechanical characterization of a composite structure is more complex than that of metallic structure
2. The design of fibre reinforced structure is difficult compared to a metallic structure, mainly due to the difference in properties in directions
3. The fabrication cost of composites is high
4. Rework and repairing are difficult

1.2. ProblemStatement:

The laminated composite beam structures are acting as most desired structure in various field of modern engineering. These are frequently used in various industries such as aerospace, construction, nuclear field, automotive, petrochemical industries, ship & rocket building due to high specific strength and stiffness of composite materials. Because of their ability to meet the design requirements of strength & stiffness, composite materials have greater importance in recent years. But during the manufacturing and service life, there will be delamination damage in composite laminate which is not visible due to embedded within the composite structures. However it reduces the strength and stiffness of the laminated structures and also effects on the vibration characteristics of the structures.

1.3objectives

a. To increase the strength and stiffness of an laminated composite beam
b. To find out the natural frequency of laminated composite beam
c. To see the effect on laminated composite beam.
    a) By changing the stacking sequence
    b) By changing the fibre orientation angle

1.4 Methodology

Analytical Solution

1. Strength and stiffness of a laminated composite beam.
2. Fundamental bending natural frequency of laminated composite beam.

II. LITERATURE REVIEW

M. Lopez-Aenlle et al. [3], studied Structural stability is one of the design requirements in laminated-glass beams and plates due their slenderness and brittleness. In this paper the equations of the classical Euler theory for buckling of isotropic monolithic beams are extended to laminated-glass beams using the effective thickness and the effective Young modulus concepts. It is demonstrated that the dependency of the effective stiffness on boundary conditions can be considered using buckling ratios of Euler theory corresponding to isotropic linear monolithic beams. The analytical predictions are validated by compressive experimental tests in simply supported beams. Fixed boundary conditions are difficult to reproduce in experimental tests due to the brittleness of the glass and for this reason fixed–fixed and fixed–pinned boundary conditions were validated using a finite element model.

Ankur A. Mistry et al. [4],in this article, epoxy specimen is manufactured, elastic modulus for matrix material i.e. Epoxy has found experimentally in universal testing machine. And in theoretical background Timoshenko beam deflection theory uses superposition of the bending and shears deflection. Modelling of composite specimen will be carried out and by finite element analysis stress strain behaviour of such a specimen is analyzed with ansys software. Thus the effect of change in volume fraction of fibers in a glass epoxy composites is analyzed theoretically, experimentally and by finite element method.

Mehdi Hajianmaleki et al. [5], this work attempts to review most of the research done in recent years (1989–2012) on the vibration analysis of composite beams. A simple classic and shear deformation model would be explained that can be used for beams with any laminate.

El Bikri Khalid et al. [6], investigate the problem of geometrically nonlinear free vibration of symmetrically and asymmetrically laminated composite beams with immovable ends can be reduced to that of isotropic homogeneous beams with effective bending stiffness and axial stiffness parameters.

Amal E. Alshorbagy et al. [7], investigate the dynamic characteristics of functionally graded beam with material graduation in axially or transversally through the thickness based on the power law. Thepresent model is more effective for replacing the non-uniform geometrical beam with axially or transversally uniform geometrical graded beam.

Li Jun et al. [8], studied dynamic finite element method for free vibration analysis of generally laminated composite beams is introduced on the basis of first order shear deformation theory.

H. Matsunaga et al. [11], Studied Natural frequencies and buckling stresses of laminated composite beams are analysed by taking into account the complete effects of transverse shear and normal stresses and rotatory inertia. By using the method of power series expansion of displacement components, a set of fundamental dynamic equations of a one-dimensional higher order theory for laminated composite beams subjected to axial stress is derived through Hamilton's principle. Several sets of truncated approximate theories are applied to solve the eigenvalue problems of a simply supported laminated composite beam.

III. DESIGNOF COMPOSITE LAMINATED BEAM

Classical Lamination Theory from Classical Plate Theory

The classical lamination theory is almost identical to the classical plate theory, the only difference is in the material properties (stress-strain relations). The classical plate theory usually assumes that the material is isotropic, while a fiber reinforced composite laminate with multiple layers (plies) may have more complicated stress-strain relations.

3.1.1 Strength Needed in More Than One Direction

Considering its light weight, a lamina (ply) of fiber reinforced composite is remarkably strong along the fiber direction. However, the same lamina is considerably weaker in all off-fiber directions. To address this issue and withstand loadings from multiple angles, one would use a lamination constructed by a number of lamina oriented at different directions.

3.1.2 Basic Assumptions of Classical Lamination Theory

Similar to the Euler-Bernoulli beam theory and the plate theory, the classical lamination theory is only valid for thin laminates (span a and $b >$ 10×thinckness t) with small displacement w in the transverse direction ($w << t$). It shares the same classical plate theory assumptions:

3.1.3 Kirchhoff Hypothesis

1. Normals remain straight (they do not bend)

2. Normals remain unstretched (they keep the same length)
3. Normals remain normal (they always make a right angle to the neutral plane)

In addition, **perfect bonding** between layers is assumed.

3.1.4 Perfect Bonding

1. The bonding itself is infinitesimally small (there is no flaw or gap between layers).
2. The bonding is non-shear-deformable (no lamina can slip relative to another).
3. The strength of bonding is as strong as it needs to be (the laminate acts as a single lamina with special integrated properties).

3.2 Selection of Carbon Epoxy Composite

Following are the features of carbon epoxy composite, the reason for which it is chosen.

1. Carbon epoxy composite gives high tensile strength, high modulus of rigidity as compared to other composites.
2. Carbon epoxy composite has unique damping characteristic.
3. Carbon epoxy composite has positive coefficient of thermal expansion i.e. tensile strength of this composite increases with temperature.
4. Carbon epoxy composite is fatigue, wear and corrosion resistant.

3.2.1Selection of Material

Basedon the advantages discussed above, the high strength and high modulus Carbon/Epoxy materials are selected for composite beam.

3.2.2Carbon Fiber(Panex 35)

A carbon fibre is a long, thin strand of material about 0.0002-0.0004 in (0.005-0.010 mm) in diameter and composed mostly of carbon atoms. The carbon atoms are bonded together in microscopic crystals that are more or less aligned parallel to the long axis of the fibre. The crystal alignment makes the fibre incredibly strong for its size. Several thousand carbon fibres are twisted together to form a yarn, which may be used by itself or woven into a fabric.Carbonfibre and matrix is used in this case in the proportion of 70:30 volume fractions.

Panex® 35 continuous carbon fibre is manufactured from polyacrylonitrile (PAN) precursor. The consistency in yield and mechanical properties that are provided by large filament count strands gives the user the ability to design and manufacture composite materials with greater confidence and allows for efficient and fast build up of carbon fibre reinforced composite structures. Material properties of Carbon fibre (Panex 35) are as follows.

Table 3.2.1 Material properties of carbon fiber (Panex 35)

Parameter	SI	US
Tensile Strength	4137 MPa	600 ksi
Tensile Modulus	242 GPa	35 msi
Electrical conductivity	0.00155 ohm-cm	0.00061 ohm-in
Density	1.81 g/cc	0.065 lb/in3
Fiber Diameter	7.2 microns	0.283 mils
Carbon content	95%	95%
Yield	270 m/kg	400 ft/lb

Weight	5.5 kg, 11 kg	12 lb, 24 lb
Spool Length	1,500 m , 3,000 m	1,640 yd, 3,280 yd

3.2.3 Epoxy resin (Araldite LY 1564/ Aradur 3486)

Epoxy resin LY1564 having low viscosity and high flexibility. The reactivity may easily be adjusted to demands through the combination of both hardeners. The long pot life of XB 3486 facilitates the production of very large industrial parts. The systems are qualified by Germanischer Lloyd.

Processing

- Pultrusion, Filament Winding
- Resin transfer moulding of small parts
- Wet lay-up

Table 3.2.2 Mechanical properties epoxy resin (LY-1564/Aradur 3486)

Sr No	Description	Unit	Typical Values
1	Appearance	Visual	Clear colourless to slightly yellow liquid
3	Density	g/cm^3	1.1-1.2
4	Viscosity	g/cm^3	1200-1400
5	Epoxy Equivalent wt	Gm./eq	180-190
1	Tensile strength	MPa	60-70
2	Tensile elongation at break	%	4.6-5.0
3	Tensile modulus	MPa	2860-3000
4	Flexural strength	MPa	118-130
5	Flexural elongation at break	%	5.5-6.5
6	Flexural modulus	GPa	2900-3050
7	Shear strength	MPa	53-58

3.2.4Material Costs

Carbon fibre cost is key to the viability of composite beam. A mean price of 2500 Rs / kg was taken for carbon fibre (250Rs / kg for glass fibre and 450Rs / kg for an epoxy resin.)

IV. MATHEMATICAL ANALYSIS OF COMPOSITE LAMINATE BEAM

A] Micromechanical Analysis of Lamina

a. Volume fraction of fibre – 0.7 (70%)
b. Volume fraction of matrix – 0.3 (30%)
c. Volume of composites – 1 (100%)

4.1 Size of the Specimen

Length (mm)	Width (mm)	Thickness (mm)
250	25	6.8

Using the formula of micromechanical analysis of lamina following properties are calculated which is shown in Table 4.3

Table 4.1 Properties of composite lamina (carbon fibre and epoxy resin)

Sr. No	Composite Properties	Symbol	Value	Unit
1	Young's Modulus(Longitudinal direction)	E_1	170.3	GPa
2	Young's Modulus(Transverse direction)	E_2	9.7	GPa
3	Major Poison's ratio	μ_{12}	0.3	-
4	Minor Poisson's ratio	μ_{21}	0.3	-
5	Shear Modulus	G_{12}	3.72	GPa
6	Ultimate longitudinal strength	$(\sigma_1^T)_{ult}$	2910	MPa
7	Ultimate transverse strength	$(\sigma_2^T)_{ult}$	25.81	MPa
8	Ultimate longitudinal compressive strength	$(\sigma_1^C)_{ult}$	83.7	MPa
9	Ultimate transverse compressive strength	$(\sigma_2^C)_{ult}$	20.21	MPa
10	Minimum fiber volume fraction	$(V_f)_{min}$	0.52	%
11	Critical fiber volume fraction	$(V_f)_{cr}$	0.53	%
12	Shear strength	τ	12.85	MPa

4.2Relationship of Compliance and Stiffness matrix to Engineering Elastic Constants of lamina

A unidirectional lamina falls under the orthotropic material category. For an orthotropic plane stress problem can be written as

$$\begin{bmatrix}\epsilon_1\\ \epsilon_2\\ \gamma_{12}\end{bmatrix}=\begin{bmatrix}S_{11} & S_{12} & 0\\ S_{12} & S_{22} & 0\\ 0 & 0 & S_{66}\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix} \qquad \text{....................} \ (4.20)$$

Where S_{ij} are the elements of the compliance matrix. Inverting the above equation gives the stress-strain relationship as

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{bmatrix}\epsilon_1\\ \epsilon_2\\ \gamma_{12}\end{bmatrix} \qquad \text{.................................} \ (4.21)$$

Where,Q_{ij} are the reduced stiffness coefficients, which are related to the compliance coefficient. The above equation 1and equation 2 shows the relationship of stress and strain through the compliance [S] and reduced stiffness [Q] matrices. However, stress and stress generally related through engineering elastic constant.

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4.2.1 Compliance and Stiffness matrix for a Carbon fibre/Epoxy lamina

A. For a 0^0 lamina

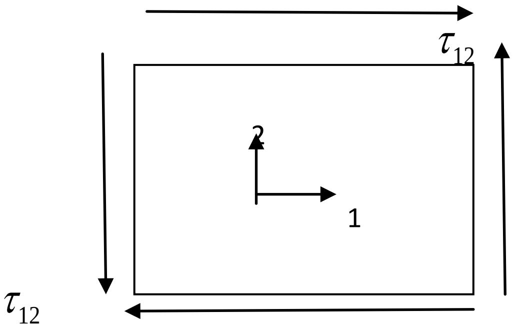

Fig. 4.4.1 Applied stresses in a unidirectional 0^0 lamina

The obtaimed values are inserted in equation 4.20

- Invert the transformed compliance matrix $[\bar{S}]$ to obtain the transformed reduced stiffness matrix $[\bar{Q}]$

$$[\bar{Q}] = [S]^{-1} = \begin{bmatrix} 5.87 \times 10^{-12} & -1.76 \times 10^{-12} & 0 \\ -1.76 \times 10^{-12} & 1.05 \times^{-10} & 0 \\ 0 & 0 & 2.68 \times 10^{-10} \end{bmatrix} \text{.................} \quad (4.30)$$

The transformed reduced stiffness matrix $[\bar{Q}]$ for 0^0carbonfiber/ epoxy ply is

$$[\bar{Q}] = \begin{bmatrix} 171.21 & 2.86 & 0 \\ 2.86 & 9.57 & 0 \\ 0 & 0 & 3.73 \end{bmatrix} 10^9 \text{ ....} \quad (4.31)$$

B. For a 20^0 lamina

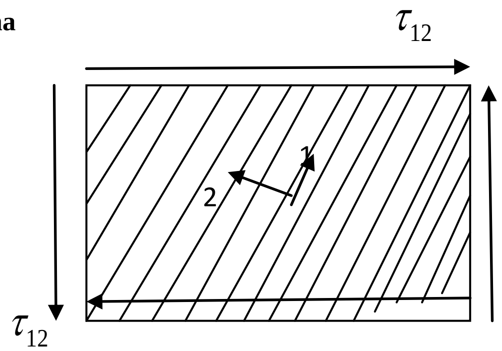

Fig. 4.4.2 Applied stresses in a unidirectional 20^0lamina

- Invert the transformed compliance matrix $[\bar{S}]$ to obtain the transformed reduced stiffness matrix $[\bar{Q}]$

$$[\bar{Q}]_{45} = [S]^{-1} = \begin{bmatrix} 3.39 \times 10^{-11} & -1.79 \times 10^{-11} & -6.97 \times 10^{-11} \\ -1.79 \times 10^{-11} & 1.07 \times 10^{-10} & 7.30 \times 10^{-12} \\ -6.97 \times 10^{-11} & 7.30 \times 10^{-12} & 2.04 \times 10^{-10} \end{bmatrix}$$

$$[\bar{Q}]_{20} = \begin{bmatrix} 127 & 18.35 & 42.7 \\ 18.35 & 12.01 & 5.84 \\ 42.79 & 5.84 & 19.31 \end{bmatrix} 10^9$$

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C. For a 45^0 lamina

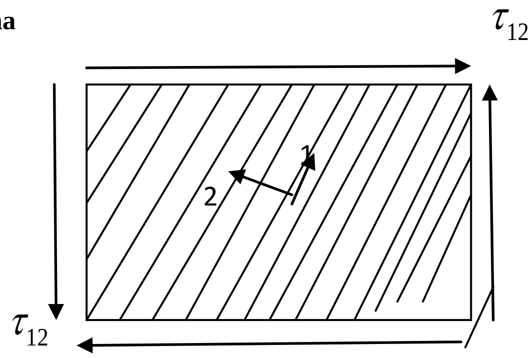

Fig. 4.4.3 Applied stresses in a unidirectional45^{0}lamina

➢ Invert the transformed compliance matrix $[\bar{S}]$ to obtain the transformed reduced stiffness matrix $[\bar{Q}]$

$$[\bar{Q}]_{45} = [S]^{-1} = \begin{bmatrix} 9.70\times10^{-11} & -4.08\times10^{-11} & -5.02\times10^{-11} \\ -4.08\times10^{-11} & 9.70\times10^{-11} & -5.02\times10^{-11} \\ -5.02\times10^{-11} & -5.02\times10^{-11} & 1.12\times10^{-10} \end{bmatrix}$$

$$[\bar{Q}]_{45} = \begin{bmatrix} 48.27 & 41.01 & 40.02 \\ 41.01 & 48.27 & 40.02 \\ 40.02 & 40.02 & 44.80 \end{bmatrix} 10^9$$

D. For a -45^0 lamina

Follow the same procedure which is used for 45^0 lamina. We get the elements of the transformed reduced compliance matrix.

E.For a 90^0 lamina

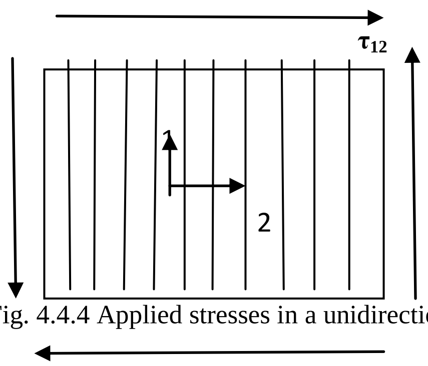

Fig. 4.4.4 Applied stresses in a unidirectional 90^{0}lamina

$$C = Cos(90) = 0$$

$$S = Sin(90) = 1$$

➢ Invert the transformed compliance matrix $[\bar{S}]$ to obtain the transformed reduced stiffness matrix $[\bar{Q}]$

$$[\bar{Q}] = [S]^{-1} = \begin{bmatrix} 1.05\times10^{-10} & -1.76\times10^{-12} & 0 \\ -1.76\times10^{-12} & 5.87\times10^{-12} & 0 \\ 0 & 0 & 2.68\times10^{-10} \end{bmatrix}$$

$$[\bar{Q}]_{90} = \begin{bmatrix} 9.57 & 2.86 & 0 \\ 2.86 & 171 & 0 \\ 0 & 0 & 3.73 \end{bmatrix} 10^9$$

B] Macro mechanical Analysis of Laminate

Macro mechanical Analysis will be developed for a laminate. A real structure however will not consist of a single lamina but a laminate consisting of more than one lamina bonded together through their thickness. First,

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lamina thicknesses are on the order of 0.005 in. (0.125mm), implying the several laminas will be required to take realistic loads. Second, the mechanical properties of a typical unidirectional lamina are severely limited in the transverse direction. If one stacks several unidirectional layers, this may be an optimum laminate for unidirectional loads. However for complex loading and stiffness requirements, this would not be desirable. This problem can be overcome by making a laminate with layers stacked at different angles for given loading and stiffness requirement. This approach increases the cost and weight of the laminate and thus it is necessary to optimize the ply angles. Moreover, layers of different composite material system may be used to develop a more optimum laminate.

$$[A] = A_{ij} = \sum_{k=1}^{3} \left[Q_{ij} \right]_k (h_k - h_{k-1})$$

A_{ij} = Extensional matrices

Inverting the extensional stiffness matrix [A], we get the extensional compliance matrix as which gives,

$$[A^*] = \begin{bmatrix} 3.32 \times 10^{-12} & -1.23 \times 10^{-12} & -1.29 \times 10^{-12} \\ -1.23 \times 10^{-12} & 2.66 \times 10^{-12} & 3.58 \times 10^{-13} \\ -1.29 \times 10^{-12} & 3.58 \times 10^{-13} & 6.11 \times 10^{-12} \end{bmatrix}$$

The coupling stiffness matrix [B] is

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{3} \left[\bar{Q}_{ij} \right]_k (h_k{}^2 - h^2{}_{k-1})$$

Also, for a symmetric laminate, the coupling matrix [B] = 0; then, from equation

The Bending stiffness matrix [D] is

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{3} \left[\bar{Q}_{ij} \right]_k (h_k{}^3 - h^3{}_{k-1})$$

Bending compliance matrix

$$[D^*] = \begin{bmatrix} 2.02 \times 10^{-12} & -2.83 \times 10^{-13} & -7.21 \times 10^{-13} \\ -2.83 \times 10^{-13} & 4.31 \times 10^{-13} & -4.74 \times 10^{-14} \\ -7.21 \times 10^{-13} & -4.74 \times 10^{-14} & 2.57 \times 10^{-12} \end{bmatrix} \frac{1}{pa - m} \quad \ldots\ldots (4.50)$$

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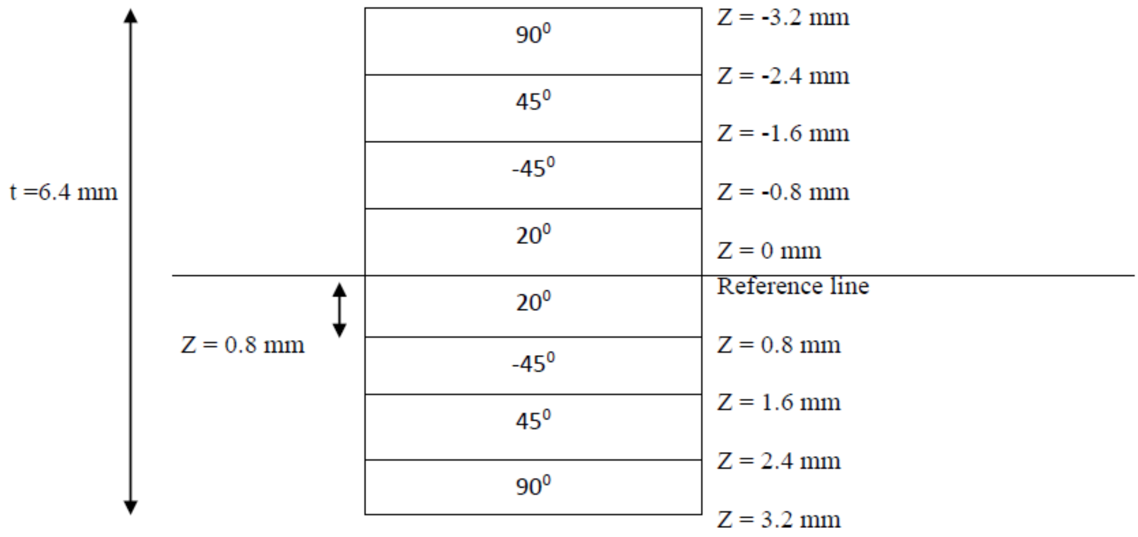

Fig 4.6 Thickness and coordinate location of the Eight-ply laminates $[90/45/-45/20]_s$

4.3 Natural frequency

Let us find out the minimum natural frequency of Composite beam, which is given by

Where P=1

$$f_{nt} = \frac{\pi P^2}{2}\sqrt{\frac{EI_X}{mL^4}} \qquad \text{..................(4.53)}$$

$$f_{nt} = \frac{\pi}{(2)}\sqrt{\frac{(47.05\times10^9)(6.55\times10^{-10})}{(0.065)(0.25^4)}}$$

$$f_{nt} = 173.05\ HZ$$

V. CONCLUSION

The following conclusions are drawn from the Present work.

- The carbon/Epoxy Composite beam has been designed to replace Glass/Epoxy Composite beam. It has been designed optimally by using classical lamination theory with the objective of increase in Strength &Stiffness, natural bending frequency of Composite laminate Beam.
- The bending natural frequency increase by decreasing the fibre orientation angle. Decreasing the angle increase the modulus in the axial direction. The layers stacking sequence has no effect on the natural frequency since there is no load applied.
- The component D_{16} corresponds to coupling between twist moment and normal curvature at axial direction has an effect on the natural frequency in applying loads to the Composite Beam. As the load increases, the natural frequency decreases

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