SPUR GEAR CRACK PROPAGATION PATH ANALYSIS AND ITS EFFECT ON GEAR MESH STIFFNESS

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ABSTRACT

Due to presence of non-uniform load distribution, local non-homogeneity of material quality and potential misalignment of gear shafts and bearings etc. cracks may occur in fillet region of spur gear teeth. Which causes failure of gear, this failure can be avoided by understanding and analysing the manner it originates. Considering tooth fracture it is necessary to study crack propagation path in spur gear. In this paper crack propagation path is predicted using Finite Element Method. Also an analytical investigation of the influence of crack propagation on gear mesh stiffness is presented. The results are important for dynamic simulation of gear transmission behaviour and simultaneously helpful for the monitoring of gearbox working condition and finding the early crack damage that may exist in gear sets.

Keywords: Crack propagation, FEA, Gear Mesh Stiffness

I.INTRODUCTION

Spur gear sets are most commonly used for power transmission in industrial applications, automobiles, aerospace and domestic equipment. Due to presence of non-uniform load distribution, local non-homogeneity of material quality and potential misalignment of gear shafts and bearings etc. cracks may occur in fillet region of spur gear teeth. The crack growth causes decrease in gear strength which affect the dynamic behaviour of gear transmission also bring out strong vibration and noise in gear system.In order to avoid the failure of gear system due to crack propagation it is necessary to diagnose gear crack in early stages using available methods. Normally, two approaches are followed: Analytical method and Finite Element Analysis (FEA). The widely used method by the researcher is FEA in which researcher have to build gear mesh models and calculate Gear Mesh Stiffness (GMS). Amongst these two methods FEA require sufficient mesh refinement and it is computationally expensive. On the other hand, an analytical method give satisfactory results and good agreement with FEA result with lower computational efforts.The Weber was first researcher who consider dividing the calculation of gear tooth deformation into three separate factors: (1) Local deformation of each tooth caused by Hertzian contact between mating teeth; (2) Basic deflection of each tooth of gear when teeth consider as non-uniform cantilever beam and tooth foundation is assumed perfectly rigid; (3) deflection of each tooth which is caused by the flexibility of the foundation when the tooth is assumed as rigid. He also proposed a strain energy method which obtain analytical expressions for each deflection based on an integration of the original shape of the tooth. In addition to the above three factors, Attia and Savage and Caldwell discussed the deflection under effect from loaded, neighbouring teeth cut in thin rims. As to the local Hertzian contact induced deformation, Cornell study and compare three typical expressions given by previous researchers: (1) an approximate Hertzian and compression approach; (2) a semi-empirical approach which is proposed by Palmgren (3) a closed form approach developed by Weber. Cornell concluded that Weber's equation gives more compatible results. According on the previous work, Yang and Sun derived an approximate formula to calculate the gear tooth local Hertzian stiffness K_h, which is in a detailed form and also it is easy to follow in the analytical procedure. Yang and Lin used a potential energy method which consider only Hertzian energy, bending energy and axial compressive energy and calculate the mesh stiffness related to these components separately. These components include Hertzian stiffness Kh, bending stiffness Kb and axial compressive stiffness Ka. Later, Tian refined his mesh model by considering shear energy and developed the expression to calculate shear stiffness Ks. Their expressions are more detailed, clear in form and easy to be programmed, and therefore will be used in this paper to calculate gear tooth stiffness. Gear teeth mesh models including fatigue cracks in the fillet region have been considered as subject of a large amount of research. Lewicki analysed the tooth crack propagation path using the FEA method and found that the crack propagation path tends to be smooth, continuous and quite straight with only a slight curve. Tian and Chaari et al. implemented a crack along the complete width of the tooth with uniform depth into their mesh models. Wu assumed that the crack only propagates in the crack depth direction and this simplifies the crack model by considering the crack paths to be straight lines which are assumed as symmetric around the tooth central line. Chen and Shao form a model with assumption that crack depth is non-uniform along the gear tooth width and at the same time crack propagates along the tooth width. Mohammed et al. studied a crack propagation scenario in which the crack extending in the depth direction and tooth width direction simultaneously. Recently, Pandya and Parey have conducted a series of studies regarding the crack behaviour in spur gear teeth and the influence on the GMS with different gear and crack parameters.All of the previous crack models intend take only plane cracks into consideration, which consider the cracks propagating either in the depth direction or in direction of the tooth width and it neglect the more typical spatial crack (three-dimensional (3D) crack) that will propagate not only in the depth direction and the direction of tooth width but also in the tooth profile direction. In this paper, modified expressions to calculate GMS based on Chen and Shao's work are proposed to account more general and reasonable approach in real-world situations.

II. CRACK PROPAGATION PATH ANALYSIS

2.1 Gear Model in Pro-E

The basic spur gear tooth geometry data was given to a tooth coordinate generation. The output defines a single tooth sector of gear. From that single tooth sector coordinate, the complete gear model was generated

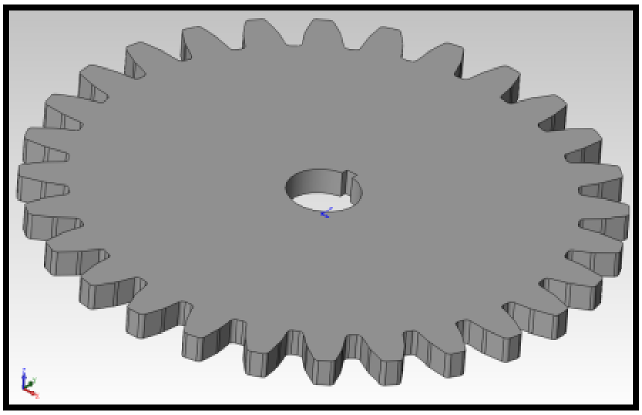


Figure 1: Isometric spur gear model using PRO-E

The gear design parameters are: Number of teeth=28; Diametric pitch=201mm; Pitch radius=44mm; Pressure angle=20°. The tooth load applied at the highest point of single tooth contact normal to the surface.

2.2 FEA Procedurein ANSYS

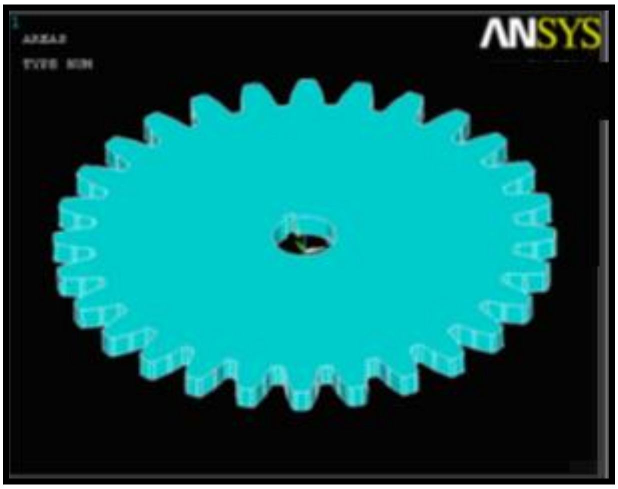


Figure 2: Finite element model for crack propagation; 8 diametral pitch, 28 teeth,445 mm pitch radius200, 20° pressure angle, with standard fillet

2.2.1. Pre-processing

Ansys which is simulation software helps to build a complete finite element model, including physical and material properties, loads and boundary conditions, and analysis the various behaviours of mechanical components and structure. Pre-processing consist of building, meshing and loading the model created.

2.2.2. Meshing

Ansys offers set of tools for automatic mesh generation with all parts of model defined, nodes, elements, restraints and loads, here the analysis part of the model is ready to begin. An analysis requires Nodal point, Elements connecting the nodal points, physical properties, material properties and Boundary conditions which consist of loads and constraint, Analysis option: how the problem will evaluated. Before generating the mesh, definition of appropriate element attributes needed. The element attributes contain Element type, Real constants, Material properties, Element coordinate system.

- Element attributes:

Element name-PLANE 82; Element shape- 2D eight node quadrilateral and 2D six node triangular elements; Nodes- I,J,K,L,M,N,O,P; Material property-EX=2e11; Poison's ratio-NUXY-0.3; Degree of freedom-UX, UY

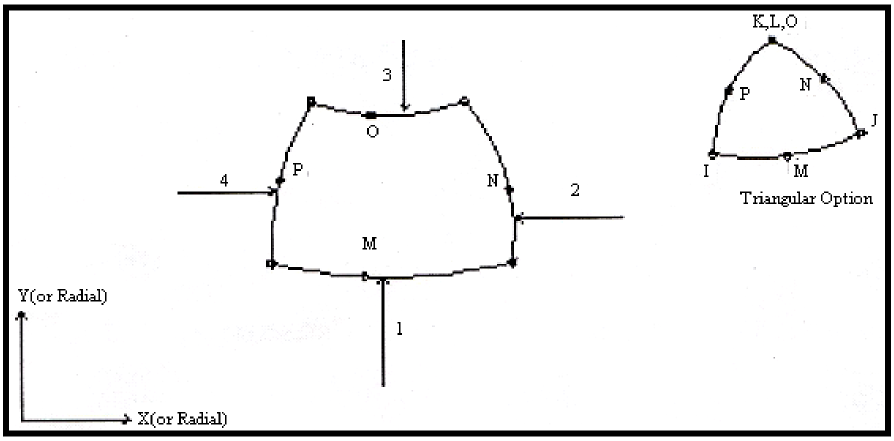


Figure 3: Element shape

2.2.3. Meshing of Gear

After defining element attributes, and meshing Control, the mesh has been generated automatically by picking the areas, which is going to mesh. In a crack model, near the crack tip node the meshing method used is "delete and fill" meshing. Six node triangular elements are used in this method.

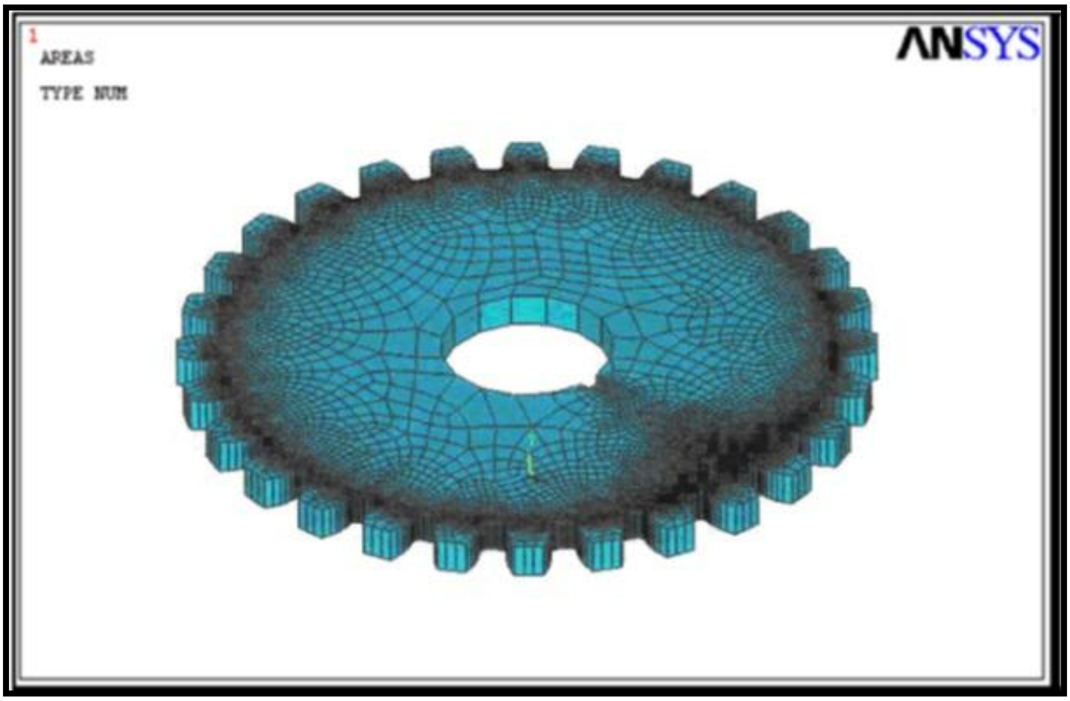


Figure 4: Spur gear meshed model

2.2.4. The Crack Modelling

The recently developed approach used is dual boundary elements to represent the crack, the model of the edge crack using this approach. In this case the modelling is extremely simple and economical. The crack is represented by two elements occupying the same physical location and each element representing of face of the crack.

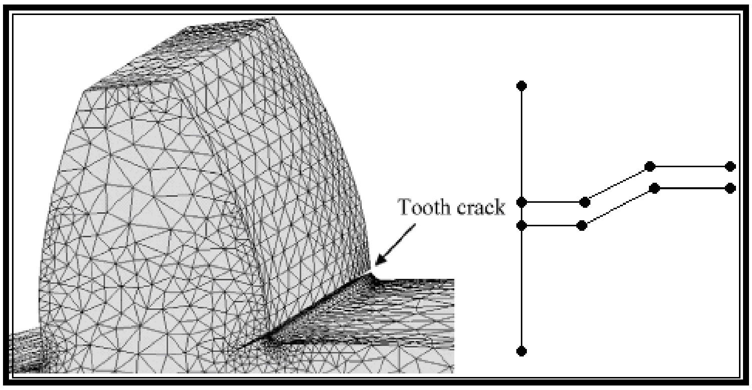


Figure 5: Dual boundary element representations of crack

2.2.5. The Direction Angle of Crack

To estimate the crack growth parameter used is stress intensity factor in crack region. Stress intensity factor values are KI = 22.5900 and KII =2.6802, finding crack direction angle calculation [θ_c]

$$\tan(\theta c/2) = \left({}^{KI}/_{4KII}\right)[1 + \sqrt{(1 + 8(KII/KI)^2)}]$$

$$\tan(\theta c/2) = \left({}^{22.590}/_{4 * 2.6802}\right)[1 + \sqrt{(1 + 8(22.590/2.6802)^2)}]$$

$$\theta c = 44.50$$

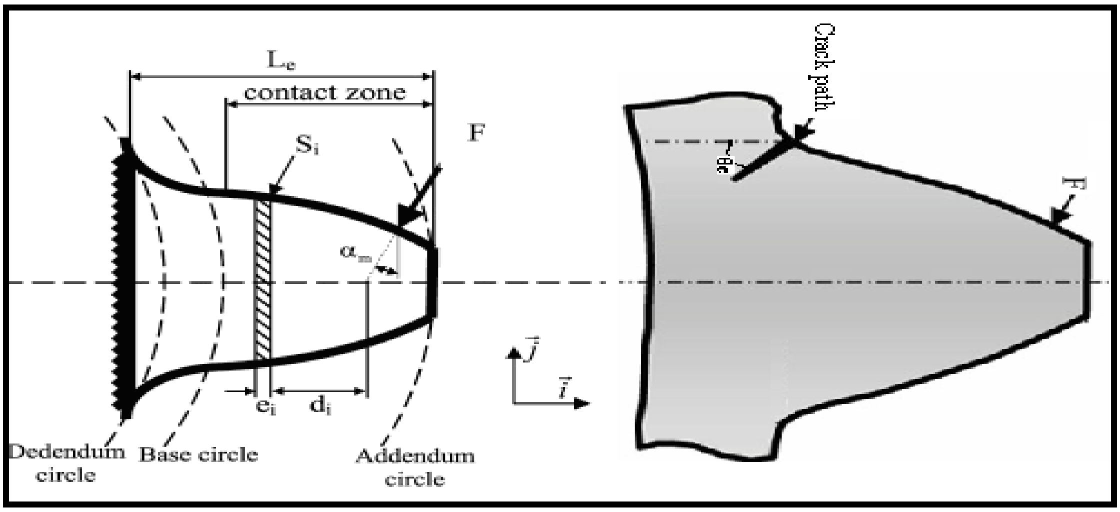


Figure 6: Geometric parameters

2.2.6. Crack-Extension Criteria

The maximum principal stress criterion states that the growthof the crack will occur in a direction perpendicular to the maximum principal stress. Hence, the local crack growth direction is determined by the condition that the local shear stress is zero.

2.2.7. Incremental crack extension analysis

The assumption made during incremental crack extension analysis is a piece wise linear distinction of the unknown crack path. The dual boundary element method is applied for each increment of the crack extension, to carry out a stress analysis of the cracked structure and the technique used for the evaluation of the stress intensity factor is the J-integral.

2.3. Simulation of Crack Growth

It provides a powerful productivity tool to evaluating the behaviour of existing cracks. The boundary element method provide several advantages in crack growth simulation because high stress gradients at the crack tip can be accurately modelled and continues re-meshing required, to simulating the crack growth.

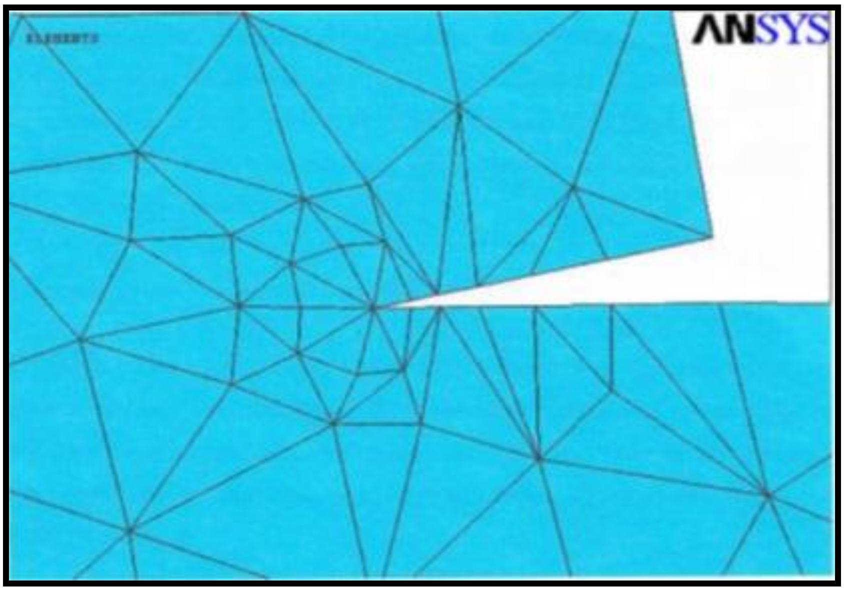
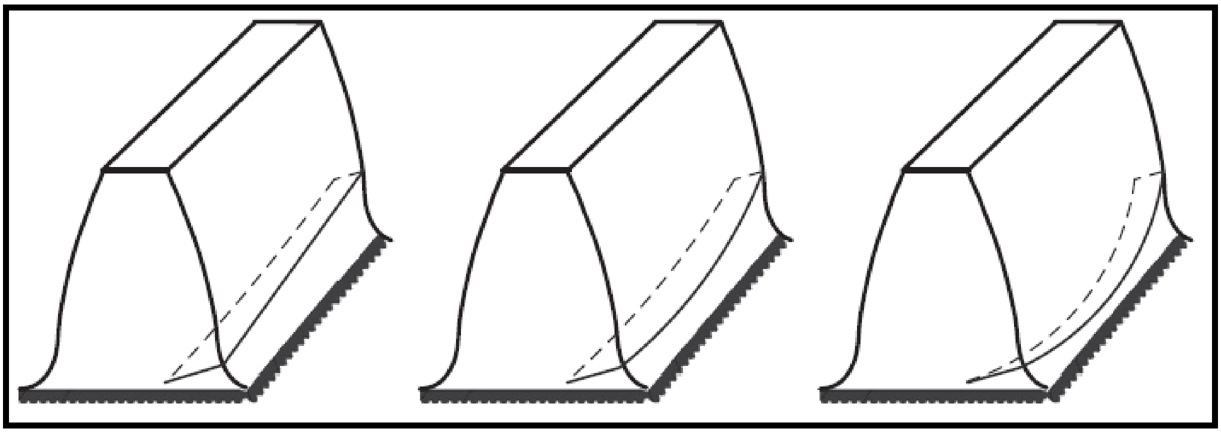


Figure 7: Crack tip model

- **Crack Propagation Procedure**

First define an initial crack by identifying coordinates of crack tip and the node of the crack mouth. Insert a 6 node triangular elements around the crack tip. Then fill the remaining surrounding area of the crack tip between the rosette and original mesh with conventional 8 node quadrilateral elements. After the initial crack is inserted in a mesh, the incremental crack extension analysis is used to simulate the crack propagation and calculate stress intensity factors, crack propagation angle. Then the places of new crack tip at the calculated angle and define crack incremental length. The model is re-meshed using the delete and fill. The procedure is repeated a number of times. In order to mixed mode crack growth an incremental type analysis is used where knowledge of both the size and direction of the crack increment extension is necessary. The crack growth algorithm incorporated in the calculation of direction angle for the crack extension. The growth paths of crack investigated by analytical method are linear, monotonous parabolic and non-monotonous parabolic respectively as shown in Figure 8. The crack growth depends on load distribution, tooth material texture and installation error of gear-shaft-bearing system.



(a) Linear **(b) Monotonous Parabolic** **(c) Non- Monotonous Parabolic**

Figure 8: Crack Growth Path

III. CALCULATION OF GMS & EFFECT OF CRACK PROPAGATION ON GMS

The gear mesh stiffness is a parameter which considers gear mesh condition from the point of engagement to disengagement. The GMS varies with time. Different gear parameters like tooth shape, number of teeth, gear tooth deflections, position of contact points and the tooth fault such as crack propagation causes the variation of gear mesh stiffness. The variation of GMS is calculated based on the variable crack intersection angle approach (v1,v2, . . . ,vn) approximation as shown in Figure 9.

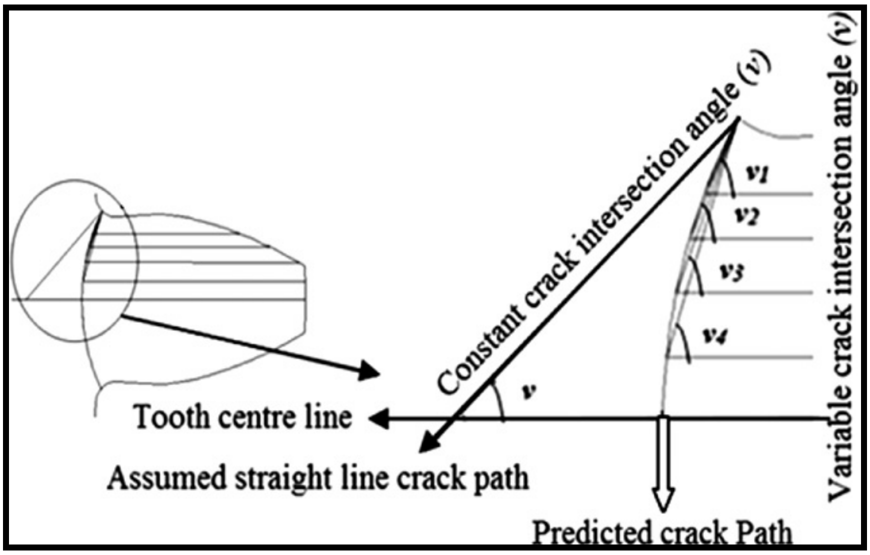


Figure 9: Predicted crack trajectory and variable crack intersection angle (v) at tooth root

The angle measured between the line joining the crack starting point to the end point of various crack length and tooth central line is the variable crack intersection angle (v), as shown in Figure 9. In the previous studies the crack path is assumed as straight line path which is at an angle of 45^{o} The predicted crack propagation path is divided into equal lengths up to half length of fully developed crack. The selected crack propagation path is divided in the steps of 10% from 0% to 50%. By observations the predicted crack path, measured crack intersection angle (ν) for different gear parameters and crack lengths are listed in Table 1. However, in case of straight crack a constant crack intersection angle of $v = 45^o$ is assumed for different gear parameters and all crack lengths. The total potential energy model of Wu et al. is acquired here for calculating total gear mesh stiffness. It consist Hertzian contact stiffness, axial compressive stiffness, bending stiffness and shear stiffness. It is also a function of crack intersection angle (v), crack length and rotation angle (θ_1) of gears in mesh. In the present study a gear pair with contact ratio 1.6456 is used for which the total gear mesh stiffness can be given by the expression for single and two teeth pair contacts as,

$$Kt = \sum_{i=0}^{2} \left(\frac{1}{\frac{1}{K_{h,i}} + \frac{1}{K_{bg,i}} + \frac{1}{K_{sg,i}} + \frac{1}{K_{ag,i}} + \frac{1}{K_{bp,i}} + \frac{1}{K_{sp,i}} + \frac{1}{K_{ap,i}}} \right) \quad \text{------------------------- (1)}$$

Where i = 1 represents the first pair of meshing teeth, i = 2 represents the second pair of meshing teeth in contact. The subscripts g represent gear and p represent pinion. The terms K_h, K_b, K_s, and K_a represent the Hertzian, bending, shear and axial compressive mesh stiffness, concise expression of which are given in Wu et al. The Hertzian and when a crack is introduced axial compressive stiffness remain the same. However, the bending and shear stiffness will change due to the appearance of the crack. To calculate the expression (1) for different cases of healthy and cracked pinion for one revolution of pinion angle (θ_1) and different crack size (L) with the expressions of the components of the total mesh stiffness as described above, MATLAB programs are used. Here the crack size is limited to half of fully developed crack for damage diagnosis at early stage. From the FEM simulation, measured crack sizes obtained and crack interaction angles (ν) given in Table 1. The variation of mesh stiffness and its percentage change from the healthy condition has been quantified and listed in Table 2. The Kt (Maximum) values in the Table 2, denotes the values of total mesh stiffness when two pair of teeth is in contact and the Kt (Minimum) denoted the single tooth contact mesh stiffness.

Table 1: Crack intersection angle's (ν) for different crack length

Division of crack length of fully developed predicted and straight crack (%)	Constanat crack intersection angle (ν) (straight crack)	Variable crack intersection angle (ν) for predicted crack path
10	45^o	58^o
20		63^o
30		69^o
40		72^o
50		76^o

Table 2: Variation of total effective mesh stiffness with change in crack length for single and double tooth pair contact gear pair

% Crack Lengh	% Change in Gear Mesh Stiffness			
	Straight Crack Path		Predicted Crack Path	
	Kt Max (kN/mm)	Kt Min (kN/mm)	Kt Max (kN/mm)	Kt Min (kN/mm)
	(% change)	(% change)	(% change)	(% change)
0%	1.055	0.5828	1.055	0.5828
10%	1.0370 (1.7%)	0.5790 (0.7%)	1.0420 (1.2%)	0.5799 (0.5%)
20%	0.9880 (6.4%)	0.5704 (2.13%)	0.9943 (5.6%)	0.5716 (1.9%)
30%	0.9349 (11.4%)	0.5598 (3.95%)	0.9379 (11.1%)	0.5605 (3.83%)
40%	0.8772 (16.9%)	0.5465 (6.23%)	0.8774 (16.8%)	0.5466 (6.2%)
50%	0.8139 (22.9%)	0.5183 (11.1%)	0.8073 (23.48%)	0.5136 (11.9%)

IV. RESULT

The first aim of the paper is crack propagation path analysis and by using FEA crack propagation path is analysed. The crack propagation path are linear, monotonous parabolic and non-monotonous parabolic by analytical method. The crack growth depends on load distribution, tooth material texture and installation error of gear-shaft-bearing system.

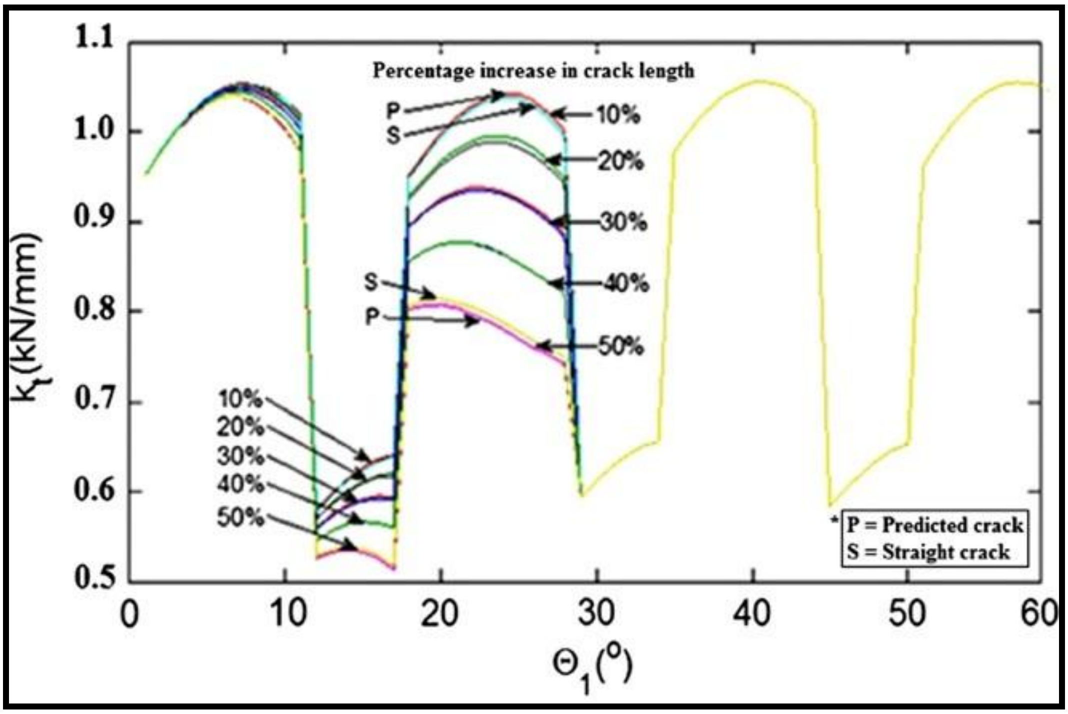


Figure 10: Variation in Gear mesh stiffness Kt, for pinion rotation, θ_1

The crack propagation produces local reduction in gear mesh stiffness in a complete rotation of the pinion. Referring to Table 2 and Fig. 10, healthy pinion, the total GMS value is 1.055 kN/mm for double tooth pair contact (Kt Maximum) and 0.5828 kN/mm for single tooth pair contact (kt Minimum). After introduction of crack GMS values changes. The percentage reduction in GMS for tooth with straight crack is from 1.7% to 16.9% is higher than the tooth with curved predicted crack from 1.2% to 16.8% for double teeth pair contact (Kt Maximum). Similar trend is observed for reduction in GMS in case of straight crack from 0.7% to 6.23% while for predicted crack during single tooth pair contact (kt Minimum) is 0.5% to 0.2%. Reversed phenomenon has been observed above 40% crack length and at 50% crack length, the percentage reduction in GMS for tooth with predicted crack is from 16.8% to 23.48% which is higher than the tooth with straight crack from 16.9% to 22.9% for double teeth pair contact (kt Maximum) and 16.2–11.9% for predicted crack and 16.23–11.1% for straight crack for single tooth pair (kt Minimum).

V. CONCLUSION

Finite element analysis and numerical studies were performed to investigate the crack propagation path and its effect on gear mesh stiffness. Along with this the percentage reduction in mesh stiffness for successive crack levels using straight and predicted crack trajectory is proposed. The important conclusions that can be obtained from the study are as follows;

1. The finite element analysis predicted that, the cracks would propagate through the tooth base region and following curved trajectory.
2. At higher levels of 40–50% crack length, reduction in gear mesh stiffness is higher for predicted curved crack path than the straight crack path.
3. At lower levels of 10–20% crack length, the difference in crack intersection angle are minimal.

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