

# **OSCILLATORY FLOW AND HEAT TRANSFER IN A VERTICAL CHANNEL WITH EQUAL WALL TEMPERATURE**

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## **ABSTRACT**

*The problem of oscillatory viscous fluid flow in a vertical channel with equal and constant wall temperature has been considered. The partial differential equations governing the flow and heat transfer are transformed to ordinary differential equations and are solved analytically using regular perturbation technique valid for small values of  $\varepsilon A$  and considering the exponential series and obtained equations can be solved.*

## **I. INTRODUCTION**

Flow heat transfer between a finite vertical parallel plates suspended in viscous fluid had been extensively investigated as one of the fundamental problem of heat transfer. Natural convection is an important heat transfer mechanism in the technology of building insulations. From the point of basic research in heat transfer, this phenomenon is being studied mainly in terms of simple model of free convection in rectangular enclosures, filled with viscous fluid. The subject of free convection in enclosures is extensive and has numerous applications in practical engineering situations.

In Natural convection, Gebhart and Pera [1] reported an analytical solution for the simultaneous heat and mass transfer problem along a vertical wall. Temperature and mass fraction of some diffusing species were imposed on the wall and the coupled mass, momentum, energy and species conservation equations were solved. The similarity method adopted by these authors made it possible to obtain solution covering both the developing and the fully developed flow regions. But, as it is now perfectly established, similarity solutions exist only under certain limited conditions. A comprehensive review of free convection heat transfer in enclosures filled with viscous fluid was presented by Ostrach [2]. Also Sastri [3] studied a problem of heat transfer in the presence of temperature dependent heat sources in the flow over a flat plate with suction. Forced and natural flows were discussed by Schlichting [4], Eckert and Drake [5] and Bansal [6]. Free convection effects on oscillatory flow of a viscous incompressible fluid past an infinite vertical plate with constant suction-I was studied by Soundalgekar [7].

There are also experimental investigations on natural convection from vertical, inclined and horizontal surfaces, covering both laminar and turbulent regions under either a constant wall temperature or a constant surface heat flux condition (Fujii and Imura [8], Shaukatullah and Gebhart [9], Yousef et. al., [10] and Siebers et. al., [11]).

However, these analytical and experimental studies were conducted under the situations of uniform thermal boundary conditions.

However, much attention has not been given to the study of oscillatory flow and heat transfer in an vertical channel with equal wall temperature even though this study is very much useful in many areas especially nuclear reactor, solar power collectors, power transformers, steam generators and others.

**II. MATHEMATICAL FORMULATION**

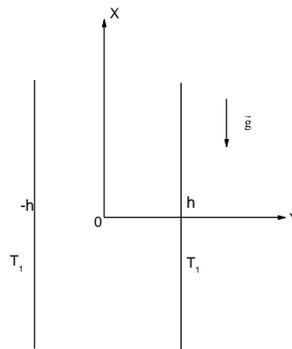


Fig. 1 Physical configuration

The physical configuration (Fig. 1) consists of two parallel vertical infinite walls maintained at equal and constant temperature  $T_1$ . The flow is assumed to be in the  $x$ -direction, which is taken along the plate in the upward direction, and  $y$ -axis is normal to it.

It is assumed that the flow is unsteady, fully developed and that fluid properties are constants. Using the conditions of the equilibrium state of the fluid and further assuming that the density of the fluid is a function of temperature alone, the equation of motion, and energy (following Stokes [12] and Umavathi and Malashetty [13]), are

$$\rho = \rho_0(1 - \beta(T - T_0)) \tag{1}$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g\beta(T - T_0) + \frac{\mu}{\rho_0} \frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = K \frac{\partial^2 T}{\partial y^2} \tag{3}$$

where  $u$  is the  $x$ -component of fluid velocity,  $v$  is the  $y$ -component of fluid velocity and  $T$  is the fluid temperature,  $T_0$  is the ambient temperature. The boundary conditions on velocity represent no-slip condition, vanishing of couple stresses at  $y = \pm h$  and that on temperature points that the plates are isothermally maintained at constant temperature  $T_1$ .

The corresponding boundary conditions are

$$u = 0 \quad \text{at} \quad y = \pm h \quad (4)$$

$$T = 0 \quad \text{at} \quad y = \pm h \quad (5)$$

Equation of continuity implies that,  $v$  is independent of  $y$ , it can be utmost a function of time alone. Hence we can write

$$v = v_0 \left( 1 + \varepsilon A e^{i\omega t} \right) \quad (6)$$

where  $A$  is real positive constant.  $\omega$  is frequency parameter and  $\varepsilon$  is small such that  $\varepsilon A \leq 1$ . The suction velocity varies periodically with time about a non-zero constant mean  $v_0$  (Sturat 1955). When  $\varepsilon A = 0$ , results are obtained for constant suction velocity. Introducing the non-dimensional parameters

$$\begin{aligned} y &= y^* h; & v &= \frac{v}{h} v^*; & t &= \frac{h^2}{\nu} t^* \\ u &= \frac{g \beta h^2 (T_1 - T_0)}{\nu} u^*; & \theta &= \frac{T - T_0}{T_1 - T_0} \end{aligned} \quad (7)$$

and for simplicity neglecting the asterisks, equations (2) and (3) becomes

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \theta + \frac{\partial^2 u}{\partial y^2} \quad (8)$$

$$\frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (9)$$

The transformed boundary conditions are

$$u = 0 \quad \text{at} \quad y = \pm 1 \quad (10)$$

$$\theta = 0 \quad \text{at} \quad y = \pm 1 \quad (11)$$

### III. SOLUTIONS

The governing equations (8) and (9) are solved subject to the boundary conditions (10) and (11). These equations are coupled partial differential equations that cannot be solved in closed form. However, it can be reduced to ordinary differential equations by assuming

$$u(y, t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + \dots \quad (12)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) + \dots \quad (13)$$

This is a valid assumption because of the choice of  $v$  as defined in equation (9) that the amplitude  $\varepsilon A \ll 1$ .

Substituting equations (12) and (13) into equations (8) to (9) and comparing the harmonic and non-harmonic terms and neglecting the higher order terms of  $\epsilon^2$ , one obtains the set of equations and which can be solved.

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