DIFFRACTION OF NORMAL SHOCK WAVE

R.S. Srivastava

Formerly of Defence Science Centre, New Delhi, India

ABSTRACT

Lighthill has considered the diffraction of normal shock wave past a small bend. Sakurai and Takayama have extended the theory of Lighthill for larger bends. In the present pressure distribution over the diffracted shock wave has been obtained using Sakurai and Takayama's theory.

Keywords: Diffraction, Normal Shock, Large bends, Pressure distribution, Singular perturbation.

I. INTRODUCTION

Lighthill (1949) considered the diffraction of normal shock wave past a small bend. Sakurai and Takayama (2005) extended the theory of Lighthill (1949) to second order terms using singular perturbation technique. The theory of Sakurai and Takayama (2005) covers shock diffraction for larger bends. Earlier Sakurai etal (2002) have obtained the pressure distribution over the diffracted shock following Lighthill's theory. In the present paper pressure distribution over diffracted shock has been obtained following Sakurai and Takayama (2005) theory by taking Mach number of the shock wave M=1.36. It may be mentioned here that Srivastava (2013) have given results for Vorticity distribution over the diffracted shock both from Lighthill's (1949) theory and Sakurai and Takayama's (2005) theory. Srivastava's (1994) book may be used for reference.

II. MATHEMATICAL FORMULATION

We have a wedge formed by two walls having a small angle δ between them. We take the coordinate (X,Y) and the origin at the leading edge of the wedge. Let $\vec{V} = (u, v)$ be the velocity, P the pressure and ρ the density. The equations for continuity, momentum and energy for the two dimensional gas flows behind the diffracted shock are

$$\frac{D\rho}{Dt} + \rho \, div \, \vec{V} = 0$$

$$\rho \frac{D\vec{V}}{Dt} + grad \, P = 0 \qquad - (1)$$

$$\frac{D}{Dt}\left(P\rho^{-r}\right) = 0$$

Where *r* being the ratio of specific heats. The following transformations are introduced.

$$x = \frac{X - U_1 t}{C_1 t}$$

$$y = \frac{Y}{C_1 t}$$

$$\frac{\vec{V}}{U_1} = (1 + u, v) \qquad \text{- (2)}$$

$$p = \frac{P - P_1}{C_1 U_1 \rho_1}$$

Where U_1 is the flow velocity C_1 is the sound speed, ρ_1 is the density and P_1 is the pressure behind the shock wave. Using small perturbation theory as proposed by Lighthill (1949) and the equations (1) and (2) gives a single partial diffraction equation in p. The equation is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1 \right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} \right) \qquad \text{- (3)}$$

The characteristics of this differential equation are tangents to the unit circle $x^2 + y^2 = 1$, so that the disturbed region is enclosed by the arc of unit circle, the diffracted shock and the wedge. The position of the shock is $x = k$, where $k = \frac{U - U_1}{C_1}$ and the coordinate of the corner is $(-M_1, 0)$ where $M_1 = \frac{U_1}{C_1}$. We takes $r = 1.4$.

Lighthill's (1949) linearized solution is for small angle δ while Sakurai and Takayama's (2005) is true for larger δ. Sakurai and Takayama used singular perturbation technique to work out their theory.

Following Lighthill (1949) and using over analysis, the pressure derivative which gives the pressure distribution over the diffracted shock is give by

$$\frac{\partial p}{\partial x_1} = \frac{C\delta}{(z_1^2 - 1)^{1/2}} \left[D - \frac{1}{(x_1 - x_0)} \right] \frac{(\alpha + \beta)(x_1 - 1)^{1/2}}{[\alpha^2 + (x_1 - 1)][\beta^2 + (x_1 - 1)]} \qquad \text{- (4)}$$

In (4) all the quantities are functions of the Mach number of the shock wave $M = \frac{U_1}{a_0}$. U is the velocity of shock, a_0 is the speed of sound ahead of shock wave, except x_1 which runs from 1 to ∞ on the diffracted shock in the transformed plane and is connected to y in the physical plane through the relation

$$y/k' = \left(\frac{x_1 - 1}{x_1 + 1} \right)^{1/2}, \qquad k' = \sqrt{1 - k^2} \qquad \text{- (5)}$$

The wall is given by $y = 0$ so that from (5) $y/k' = 0$ on the wall. For this to be true $x_1 = 1$ from (5). Further at the inter section of the unit circle and shock wave $y = k'$, so that at this point $y/k' = 1$. From this to be true $x_1 \to \infty$ from (5).

Sakurai and Takayama (2005) have extended the work of Lighthill (1949) to higher δ by considering second order terms through singular perturbation technique. Sakurai and Takayama (2005) assumed *y* on the diffracted shock and computed corresponding $\bar{y}$ from their extended theory and then computed $\bar{x}_1$. The relationship between $\bar{y}$ and $\bar{x}_1$ is same as given by (5) in which *y* is replaced by $\bar{y}$ and x_1 by $\bar{x}_1$. The new $\bar{y}$ and $\bar{x}_1$ are used to calculate pressure distribution from equation (4). The modified results of Sakurai and Takayama (2005) which are required for calculation are given below

$$\bar{y} = \sqrt{r^2 - k^2}, \quad r = \xi + \delta r_1, \quad \xi = \sqrt{y^2 + k^2}$$

$$r_1 = \kappa(\phi)\bar{R}\log\bar{R}, \quad \bar{R} = \left[\rho^2 + 2\frac{\bar{X}_0 k\rho}{\xi} + X_0^2\right]^{1/2}$$

$$\rho = 1 - \frac{\sqrt{1-\xi^2}}{\xi}, \quad \phi = \tan^{-1}\frac{y}{M_1 + k}$$

$$\kappa(\phi) = \frac{1}{\pi}\cdot\frac{M_1^4}{1-\sqrt{1-M_1^2}}\cdot\frac{1}{\cos\phi\cos 2\phi}\cdot\left[1 + \frac{r+1}{2}\cdot\frac{M_1^2}{1-M_1^2}\cos 2\phi\right]$$

Where ϕ in $\kappa(\phi)$ is a variable.

$$X_0^2 = 1 - \frac{\sqrt{1-M_1^2}}{M_1}$$

Where *y* is the *y* coordinate on the diffracted shock, ξ is the strained variable and other variables are connected within themselves.

III. NUMERICAL SOLUTION

In order to obtain pressure distribution over diffracted shock for higher bends x_1 in equation in (4) should be replaced by $\bar{x}_1$ and should be integrated to give the pressure distribution. The pressure *p* is known at $x_1 = \infty(\bar{x}_1 = \infty)$, the point at the intersection of shock and unit circle, the pressure at other points is obtained by integrating equation (4) between the intervals. The points chosen for interval are $y/k' = 0$, $y/k' = 0.25$, $y/k' = 0.50$, $y/k' = 0.75$, $y/k' = 1$. The equation (4) and (5) are used together to get the results. The following table gives the results after integration. The table is for y/k' versus $-p/k\delta$.

The Mach number of the shock wave $M = 1.36$.

Table

y/k'	0	0.25	0.50	0.75	1
$-p/k\delta$	1.87	1.80	1.69	0.58	0

The table shows that $-p/k\delta$ is maximum at $y/k' = 0$ i.e. at point of intersection of wall and shock. From there $-p/k\delta$ falls over the diffracted shock and attains the value zero at $y/k' = 1$ i.e. at the point of intersection of shock and unit circle. There is slow decrease of the pressure over the diffracted shock from $y/k' = 0$ to $y/k' = 1$.

Referring to the paper of Sakurai et al (2002), one finds that the pressure distribution over the diffracted have lower values compared to the values presented here for higher δ.

IV. CONCLUSION

The problem was earlier treated by Sakurai et al (2002) for smaller bend following Lighthill's (1949) theory. The trend of pressure distribution continuous to similar to that given by Sakurai et al (2002) of course the pressure is higher for larger δ. This may be due to the larger bend. The problem is important for aeronautical engineers and may be useful in aircraft design as well.

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