A THREE LAYER SUPPLY CHAIN INVENTORY MODEL WITH IMPERFECT QUALITY PRODUCTION SYSTEM

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ABSTRACT

In this paper, we have developed a three layer supply chain inventory model with imperfect quality products. Supplier, manufacturer and retailer are the members of the supply chain where supplier produces the raw material and delivers it to the manufacturer. The defective items produced during this period are sold at a single lot with cheaper rate in another market. Manufacturer produced the items and sold them to the retailer in multiple deliveries. Cost of Idle times at supplier, manufacturer and retailer are also considered. A mathematical formulation is presented to find the optimal replenishment policy. Finally numerical analysis is also given to illustrate the problem.

Keywords: Imperfect Production, Inventory, Secondary Market, Three Layer Supply Chain, Imperfect Production

I. INTRODUCTION

Management scientists, researchers as well as practitioners in manufacturing industries have emphasized to develop production inventory control system in supply chain management. The purpose of the management is always to coordinate the business process and activities of the channel members to improve overall performance of the chain. Due to competitive framework, firms tend to emphasize certain competitive dimensions and develop manufacturing capabilities to reach the selected dimensions to improve their market position. The competitive dimensions are cost, quality, delivery and flexibility. These dimensions relate to production process and control, technology, capacity, planning, etc. In competitive marketing environment, every corporation keeps the brand image regarding quality issue with fair prices to capture the market.

In traditional production and order level model, all items are perceived to be perfect quality. It is rational to all enterprises that all items are not perfect, a certain percent of the products are non-conforming quality. These non-conforming quality items may be reworked at a cost or sold at reduced price. Generally, in collaborating system, the defective items are sent back at reduced price to the member where it was purchased. To create an integrated supply chain management the earliest effort was made by Oliver and Webber[1]. They developed a production, distribution and inventory planning system that integrated three supply chain segments comprised of supply, storage/location and customer demand planning. Khouja and Mehrez [2] assumed three coordination mechanisms between the members of the supply chain and demonstrated that some coordination mechanisms lead to significant reduction in total cost. Banerjee and Kim [3] studied the integrated inventory models in which the vendor and the buyer coordinate their production and ordering policies, in order to reduce the joint inventory costs. Hill [4] provided a global optimal shipment policy for single vendor and single buyer integrated problem, combining the equal shipment size and policies. Lee and Wu [5] developed a study on inventory replenishment policies in a two-echelon supply chain system. Ahmed et. al [6] have coordinated a two level supply chain in which they considered production interruptions for restoring of the quality of the production process. Singh [7] assumed optimal ordering policy for decaying items under inflation. Singh et. al [8] worked on supply chain model with stochastic lead time under imprecise partially backlogging for expiring items. Kumar et al. [9] proposed a three echelon supply chain inventory model for deteriorating items with limited storage facility. Singh et. al.[10] also developed a three stage supply chain model under fuzzy random demand and production rate with imperfect production process. Chung et. al. [11] developed an inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit.

This research assesses the impact of production run time, delivery time, idle time of the systems and inventory level. It is common to all industries that a certain percent of produced/ordered items are non-conforming (imperfect) quality. In this paper we have assumed that all produced items are not perfect, a certain percent of the products is imperfect. These imperfect items are sold at a reduced price in another market.

II. ASSUMPTIONS AND NOTATIONS

The following assumption and notation are considered to develop the model.

Assumptions

1. Joint effect of supplier, manufacturer, and retailer is considered in a supply chain management.
2. Supplier produced the item with constant rate P_s unit per unit time, while manufacturer produces P_m units per unit time with $P_s > P_m$.
3. Idle cost of suppliers, manufacturer and retailer are taken into account.
4. The defective items exist in lot size (Q). Also assume that percentage defective random variable (α) is uniformly distributed with its $p.d.f.\, as\; E[\alpha]=\int_a^b \alpha\; f(\alpha)\,d\alpha, \quad 0<a<b<1$

Notations

P_s Constant production rate for the suppliers

α Percentage of defective items

P_m Demand rate or production rate for the manufacturer.

d_r Constant demand rate for the retailer

d_c Constant demand rate of customer

C_s Production cost of unit item for suppliers

C_m Selling price of unit item for suppliers which is also purchase cost for manufacturer.

C_r Selling price of unit item for manufacturer which is also purchase cost for retailers.

C_R Selling price for retailers

t_s Production time for suppliers

T_s Cycle length for Suppliers

T_R Time duration in which order is supplied by manufacturer or retailer

T Total time for supply chain

h_s Holding cost per unit per unit time for suppliers.

h_m Holding cost per unit per unit time for manufacturer.

h_r Holding cost per unit per unit time for retailers.

A_s Ordering cost for suppliers.

A_m Ordering cost for manufacturer.

A_r Ordering cost for retailers.

id_s Idle cost per unit time for suppliers

id_m Idle cost per unit time for manufacturer

id_r Idle cost per unit time for retailers

n Number of cycle for retailers

r Number of cycle where manufacturer stop the production

III. MATHEMATICAL FORMULATION OF MODEL

This section examines the profit of integration of the lot sizing policies by determining a common economic policy using the total profit for each party. Fig. 1 depicts the behaviour of inventory levels for supplier, manufacturer and the retailer, based on the above notation and assumptions.

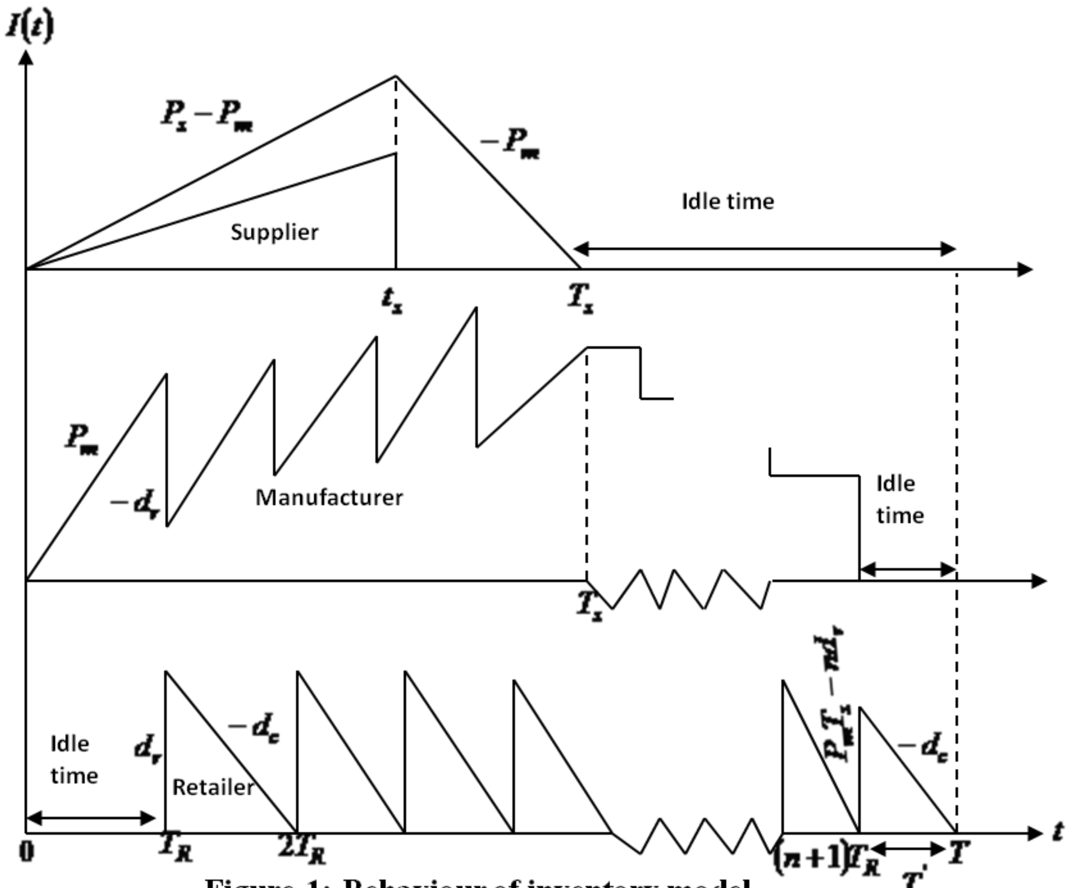

Figure-1:-Behaviour of inventory model.

3.1 Formulation of supplier's individual average profit

Differential equation for the supplier shown in Fig.2 in $[0,T]$ is given by

$$\frac{dI_s(t)}{dt}=\begin{cases}(1-\alpha)P_s-P_m & ,0\le t\le t_s\\ -P_m & ,t_s\le t\le T_s\\ 0 & ,T_S\le t\le T\end{cases}$$

$$\frac{dI_{imp}(t)}{dt}=\alpha P_s \quad , \quad 0\le t\le t_s$$

with the boundary condition $I_s(0)=0=I_s(T_s)$

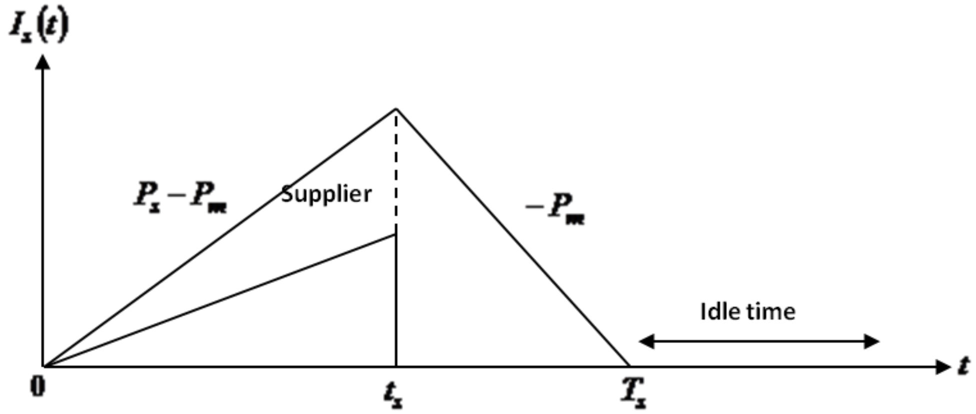

Figure-2: Inventory level of supplier

Solutions of the above differential equations are given by

$$I_s(t) = \begin{cases} \{(1-\alpha)P_s - P_m\}t & , \quad 0 \le t \le t_s \\ P_m(T_s - t) & , \quad t_s \le t \le T_s \end{cases}$$

$$Q = (1-\alpha)P_s t_s = P_m T_s \quad and \quad T_s = \frac{(1-\alpha)P_s t_s}{P_m}$$

$$I_{imp}(t) = \alpha P_s t \qquad , \quad 0 \le t \le t_s$$

Holding cost of supplier

$$= h_s\left[\int_0^{t_s} \{(1-\alpha)P_s - P_m\}t\, dt + \int_{t_s}^{T_s} P_m(T_s - t)\, dt\right]$$

$$= h_s\left[\{(1-\alpha)P_s - P_m\}\frac{t_s^2}{2} + \frac{P_m}{2}(T_s - t_s)^2\right]$$

$$= \frac{h_s(1-\alpha)t_s^2 P_s}{2}\{(1-\alpha)P_s - P_m\}$$

Holding cost of imperfect items

$$= h_{imp}\int_0^{t_s} \alpha P_s t\, dt = h_{imp}\frac{\alpha P_s t_s^2}{2}$$

The idle cost of supplier

$$= id_s\left\{T_R + (1-\alpha)P_s t_s\left(\frac{1}{d_c} - \frac{1}{P_m}\right)\right\}$$

Total production cost

$$= C_s P_s t_s$$

ISBN : 978-93-86171-07-8

Total selling price (Primary market) $= C_m P_m t_s = C_m(1-\alpha)P_s t_s$

Total selling price (Secondary market) $= C_{m1}\alpha P_m t_s$

Ordering cost $= A_s$

Total profit of supplier per unit time is given by

$TP_s = \dfrac{1}{T}$ [total sale revenue-(production cost + total holding cost+ idle cost+ ordering cost)]

$$= \frac{1}{T}\left[\begin{array}{l} \{C_m(1-\alpha)+C_{m1}\alpha - C_s\}P_s t_s - h_s \dfrac{(1-\alpha)t_s^2 P_s}{2}\{(1-\alpha)P_s - P_m\} \\ -h_{imp}\dfrac{\alpha P_s t_s^2}{2} - id_s\left\{T_R + (1-\alpha)P_s t_s\left(\dfrac{1}{d_c} - \dfrac{1}{P_m}\right)\right\} - A_s \end{array}\right]$$

3.2 Formulation of manufacturer individual average profit

Inventory level of manufacturer shown in fig.-3 in [0,T] is given by

$$I_m(t) = \begin{cases} P_m t & ,0 \le t \le T_R \\ P_m t - id_r & ,iT_R \le t \le T_R, i = 1,2,\dots\dots\dots\dots,(r-1) \\ P_m t - rd_r & ,rT_R \le t \le T_s \\ P_m T_s - rd_r & ,T_s \le t \le (r+1)T_R \\ P_m T_s - id_r & ,iT_R \le t \le (i+1)T_R, i = (r+1),(r+2),\dots\dots\dots\dots,(n-1) \\ P_m T_s - nd_r & ,nT_R \le t \le (n+1)T_R \\ 0 & ,(n+1)T_R \le t \le T \end{cases}$$

with the boundary condition $I_m(0) = 0$, and $I_m(iT_R + 0) = I_m(iT_R) - d_r$

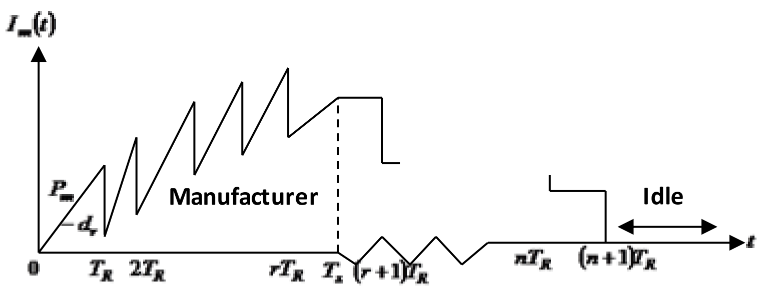

Figure-3: Inventory level of

Holding cost of manufacturer

$$= h_m \left\{ \begin{array}{l} \int_0^{T_R} P_m t \, dt + \sum_{i=1}^{(r-1)} \int_{iT_R}^{(i+1)T_R} \left(P_m t - id_r\right) dt + \int_{rT_R}^{T_S} \left(P_m t - rd_r\right) dt \\ + \int_{T_S}^{(r+1)T_R} \left(P_m T_s - rd_r\right) dt + \sum_{i=(r+1)}^{(n-1)} \int_{iT_R}^{(i+1)T_R} \left(P_m T_s - id_r\right) dt + \int_{nT_R}^{(n+1)T_R} \left(P_m T_s - nd_r\right) dt \end{array} \right\}$$

$$= h_m \left\{ nP_m T_s T_R - \left(\frac{n^2 + n - 2r - 2}{2} \right) d_r T_R - \frac{(1-\alpha) P_s^2 t_s^2}{2P_m} \right\}$$

The idle cost of manufacturer $= id_m \left\{ \frac{(1-\alpha)P_s t_s - nd_r}{d_c} \right\}$

Total purchase cost $= C_m P_m T_s$

Total production cost for the manufacturer $= C_p P_m T_s$

Total selling price $= C_r P_m T_s$

Ordering cost $= A_m$

Total profit of manufacturer per unit time is given by

$TP_m = \frac{1}{T}$ [total sale revenue-(total purchase cost + total production cost + total holding cost+ idle cost+ ordering cost)]

$$= \frac{1}{T} \left[\begin{array}{l} \left(C_r - C_m - C_p\right) P_m T_s - h_m \left\{ nP_m T_s T_R - \left(\frac{n^2 + n - 2r - 2}{2} \right) d_r T_R - \frac{(1-\alpha) P_s^2 t_s^2}{2P_m} \right\} \\ - id_m \left\{ \frac{(1-\alpha)P_s t_s - nd_r}{d_c} \right\} - A_m \end{array} \right]$$

3.3 Formulation of Retailer individual average profit

Inventory level of retailer Shown in fig.4 in [0,T] is given by

$$I_R(t) = \begin{cases} d_c (T_R - t) & , iT_R \le t \le (i+1)T_R \\ P_m T_s - nd_r - d_c t & , (n+1)T_R \le t \le T \end{cases}$$

With the boundary condition $I_R\{(n+1)T_R\} = 0$ and $I_R(T) = 0$

We have $T' = \frac{P_m T_s - nd_r}{d_c}$ and $T = nT_r + T' = nT_r + \frac{P_m T_s - nd_r}{d_c}$

Holding cost of retailer

$$= h_r \left\{ n\int_0^{T_R} d_c (T_R - t)dt + \int_0^{T'} (P_m T_s - nd_r - d_c t)dt \right\}$$

$$= \frac{h_r}{2}\left\{ \frac{P_m^2 T_s^2}{d_c} - 2nP_m T_s T_R + n(n+1)T_R d_r \right\}$$

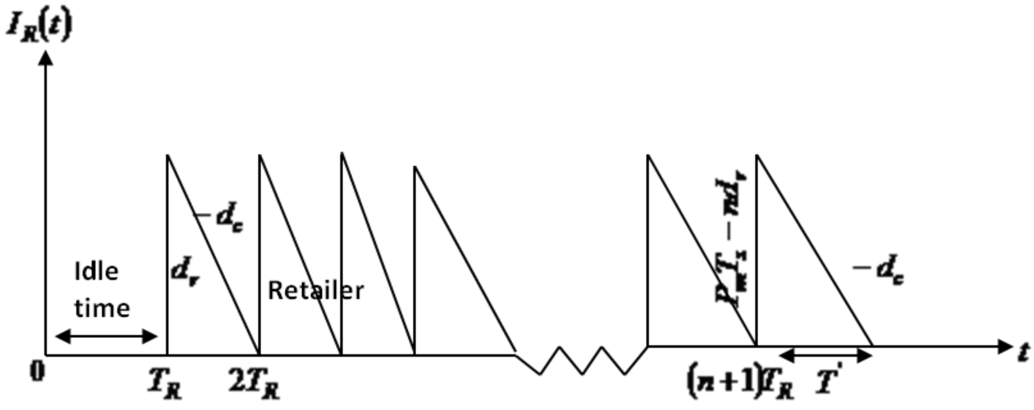

Figure-4: Inventory level of retailer.

The idle cost of retailer $= id_r T_R$

Total purchase cost $= C_r P_m T_s$

Total selling price $= C_R P_m T_s$

Ordering cost $= A_r$

Total profit of retailer per unit time is given by

$$TP_R = \frac{1}{T}\text{[total sale revenue-(total purchase cost + holding cost+ idle cost+ ordering cost)]}$$

$$= \frac{1}{T}\left[(C_R - C_r)P_m T_s - \frac{h_r}{2}\left\{ \frac{P_m^2 T_s^2}{d_c} - 2nP_m T_s T_R + n(n+1)T_R d_r \right\} - id_r T_R - A_r \right]$$

IV. TOTAL PROFIT IN SUPPLY CHAIN

Total profit of supply chain inventory model per unit time is given by

$TP =$ Total profit of supplier per unit time +Total profit of manufacturer per unit time +Total profit of retailer per unit time

$$= TP_s + TP_m + TP_R$$

$$=\frac{1}{T}\left[\begin{array}{l}\{C_m(1-\alpha)+C_{m1}\alpha-C_s\}P_st_s-h_s\frac{(1-\alpha)t_s^2P_s}{2}\{(1-\alpha)P_s-P_m\}-id_s\left\{T_R+(1-\alpha)P_st_s\left(\frac{1}{d_c}-\frac{1}{P_m}\right)\right\}\\ -h_{imp}\frac{\alpha P_st_s^2}{2}-A_s+(C_r-C_m-C_p)P_mT_s-h_m\left\{nP_mT_sT_R-\frac{n^2+n-2r-2}{2}T_Rd_r-\frac{(1-\alpha)P_s^2t_s^2}{2P_m}\right\}-A_m\\ -id_m\left\{\frac{(1-\alpha)P_st_s-nd_r}{d_c}\right\}+(C_R-C_r)P_mT_s-\frac{h_r}{2}\left\{\frac{P_m^2T_s^2}{d_c}-2nP_mT_sT_R+n(n+1)d_rT_R\right\}-id_rT_R-A_r\end{array}\right]$$

$$\textit{Since}\quad d_r=d_cT_R\quad \textit{and}\quad T_s=\frac{(1-\alpha)P_st_s}{P_m}\quad \textit{then}$$

$$TP=\frac{1}{T}\left[\begin{array}{l}\{C_{m1}\alpha-C_s\}P_st_s-h_s\frac{(1-\alpha)t_s^2P_s}{2}\{(1-\alpha)P_s-P_m\}-id_s\left\{T_R+(1-\alpha)P_st_s\left(\frac{1}{d_c}-\frac{1}{P_m}\right)\right\}\\ -h_{imp}\frac{\alpha P_st_s^2}{2}+C_p(1-\alpha)P_mt_s-h_m\left\{n(1-\alpha)P_mt_sT_R-\frac{n^2+n-2r-2}{2}T_Rd_r-\frac{(1-\alpha)P_s^2t_s^2}{2P_m}\right\}\\ -id_m\left\{\frac{(1-\alpha)P_st_s-nd_cT_R}{d_c}\right\}+C_R(1-\alpha)P_mt_s-\frac{h_r}{2}\left\{\frac{(1-\alpha)^2P_m^2t_s^2}{d_c}-2n(1-\alpha)P_mt_sT_R+n(n+1)d_cT_R^2\right\}\\ -id_rT_R-A_s-A_m-A_r\end{array}\right]$$

Total expected profit of the chain is given by

$$E[TP]=\frac{1}{T}\left[\begin{array}{l}\{C_{m1}E[\alpha]-C_s\}P_st_s-h_s\frac{(1-E[\alpha])t_s^2P_s}{2}\{(1-E[\alpha])P_s-P_m\}\\ -id_s\left\{T_R+(1-E[\alpha])P_st_s\left(\frac{1}{d_c}-\frac{1}{P_m}\right)\right\}-h_{imp}\frac{E[\alpha]P_st_s^2}{2}+C_p(1-E[\alpha])P_mt_s\\ -h_m\left\{n(1-E[\alpha])P_mt_sT_R-\frac{n^2+n-2r-2}{2}T_Rd_r-\frac{(1-E[\alpha])P_s^2t_s^2}{2P_m}\right\}\\ -id_m\left\{\frac{(1-E[\alpha])P_st_s-nd_cT_R}{d_c}\right\}+C_R(1-E[\alpha])P_mt_s\\ -\frac{h_r}{2}\left\{\frac{(1-E[\alpha])^2P_m^2t_s^2}{d_c}-2n(1-E[\alpha])P_mt_sT_R+n(n+1)d_cT_R^2\right\}-id_rT_R-A_s-A_m-A_r\end{array}\right]$$

V. NUMERICAL ANALYSIS

We consider the values of parameters in appropriate units as follows:

$$C_s = 20,\quad C_m = 51.5,\quad C_{m1} = 25\quad C_r = 80.5,\quad C_R = 100,\quad C_p = 20,\quad \alpha = 0.02,\quad h_s = 0.75,$$
$$h_{imp} = 0.5,\quad h_m = 0.75,\quad h_r = 1,\quad P_s = 2000,\quad P_m = 1500,\quad id_s = 1,\quad id_m = 2,\quad id_r = 2.5,$$
$$d_c = 1000,\quad A_s = 500,\quad A_m = 3000, A_r = 6500,\quad n = 10,\quad r = 6$$

Where $E(\alpha) = \int_a^b \alpha f(\alpha) d\alpha$ percentage defective random variable α with p.d.f.

$f(\alpha) = \begin{cases} 25 & ,0 \le \alpha \le 0.04 \\ 0 & ,elesewhere \end{cases}$,therefore $E(\alpha) = 0.02$ and the total profit of the supply chain is

$$TP = 684.38,\quad TP_s = 480.33,\quad TP_m = 142.36,\quad and\quad TP_R = 61.69.$$

VI. CONCLUSIONS

In this paper, we have derived a three-layer supply chain involving supplier, manufacturer and retailer with imperfect production. It is assumed that the cycle time at each stage is equal. The cost of idle time of supplier and manufacturer is also considered. At each stage, the defective items at supplier level are sold to another market at one lot with lower price. Finally, the supply chain profit function, combining the profit of supplier, manufacturer and retailer, is maximized.

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ISBN : 978-93-86171-07-8

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