Nonexpansive Mappings on Fixed Point Set In Metric Space

Sonia

Research Scholar, Singhania University

Abstract

This approach of mapping extends within (ZF) to finite commutative families of nonexpansive mappings. Again, let (M,d) be a bunded metric space, and let ∑ be a compact convexity structure on M. A subset A of M is said to be admissible if A =cov(A), where here we take cov( ) { : A B B = I is a closed ball containing A}. Let φ( ) M denote the set of all admissible subsets of M.

Key words: Non-Linear, Consequence,

Introduction Piers Bohl, a Latvian mathematician, applied topological methods to the study of differential equations. In 1904 he proved the three-dimensional case of our theorem, but his publication was not noticed. It was Brouwer, finally, who gave the theorem its first patent of nobility. His goals were different foundations of mathematics, especially mathematical logic and topology. The ensuing discussions convinced Brouwer of the importance of a better understanding of Euclidean spaces, and were the origin of a fruitful exchange of letters with Hadamard. H. Freudenthal comments on the respective roles as follows: "Compared to Brouwer's revolutionary methods, those of Hadamard were very traditional, but Hadamard's participation in the birth of Brouwer's ideas resembles that of a midwife more than that of a mere spectator." Brouwer's approach yielded its fruits, and in 1910 he also found a proof that was valid for any finite dimension, as well as other key theorems such as the invariance of dimension,

Review of Literature

Authors have made extensive research in the direction, which has led to many new results in the study of fixed point theory with applications in differential inclusion, economics, and related topics. Many of the most important nonlinear problems of applied mathematics reduce to finding solutions of nonlinear functional equations which can be formulated in terms of finding the fixed points of a given nonlinear mapping of an infinite dimensional function space X into itself. For mappings satisfying compactness conditions, a general existence theory of fixed points based upon topological arguments has been constructed over a number of decades. More recently, there has begun the systematic study of fixed points of various classes of noncompact mappings, some of which are described in the Discussion below

Methods: Let (M,d) be a bunded metric space, and let ∑ be a compact convexity structure on M. A subset A of M is said to be admissible if A =cov(A), where here we take cov( ) { : A B B = I is a closed ball containing A}. We shall let φ( ) M denote the set of all admissible subsets of M. Then, since ∑ congtains the closed balls of M it follows that φ( ) M is itself a compact convexity structure on M. We say that φ( ) M is normal if r A diam A ( ) ( ) < for each A M ∈φ( ) with diam A( ) 0 > . The following is the key to the structure of the fixed point sets of nonexpansive mappings. We now describe several fundamental properties of 1-local retracts. The proof of the first is routine and the second is immediate. Proposition 1. If M is a metric space for which φ( ) M is compact, then M is complete. Propositin 2. If M is a metric space for which φ( ) M is compact, and if { } An is a descending sequence of sets in φ( ) M for which lim ( ) 0 n n →∞ diam A = , then { } IA z n = The following technical proposition collects several additional properties needed in this study. Proposition 3. Let M be ametric space and let A be a nonempty subset of M. then:

cov( ) { (; ( )) : ; A B x r A x M = ∈ I x ( ) (cov( )) x x r A r A = for every x M∈ ; r A r A (cov( )) ( ); ≤ d diam A diam A (cov( )) ( ) = ( )) A B z diam A ⊆ .

References 1] M. Aamri, A. Bassou, and D. E. Moutawakil, Common fixed points for weakly compatible maps in symmetric spaces with application to probabilistic spaces, Appl. Math. E-Notes, 5(2005), 171-175.

2] M. Aamri and D. E. Moutawakil, Common fixed points under contractive conditions in symmetric spaces, Appl. Math. E-Notes, 3(2003), 156-162.

3] S. Czerwik, Nonlinear set-valued contraction mappings in b-metric spaces, Atti Sem. Mat. Fis. Univ. Modena, 46 no. 2 (1998), 263-276. MR1665883 (99j:54043).

4] P. Z. Daffer and H. Kaneko, Multi-valued f-contractions mappings, Boll. Un. Mat. Ital. A, 8 no.7 (1994), 233-241.