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PRIME PROBLEM IN GAME THEORY – BROWN'S ALGORITHM

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ABSTRACT

The paper is intended to examine a row and column both dominance prime game with Brown's Algorithm. A 10 x 10 prime game is considered on dominance strategy. Considerable conclusions are drawn. The relations among the Lower bounds and/or Upper bounds of this prime game at different stages are calculated.

Keywords: Game Theory, players, strategy, Pay-off matrix, optimal solution, Lower bound, Upper bound

AMS Classification: **91A05,91A18,91A43, 91A90**

I.INRTRODUCTION

Brown's algorithm is one of the best tool to solve any type of game in a scientific manner. Influences of each components of Player A can be observed keenly on each component of other player B. The Optimum mixed strategies may help us to evaluate any game effectively. Some games are very typical and the evaluation may have complexity to solve the game. In such cases, Brown's algorithm will classify the exact nature of the game.

In 1979,Billy E.Gillett[1] discussed many methods to evaluate many problems in game theory. Many applications in operations research (OR) are introduced by Levin and Desjardins [2] in 1970. Many mathematicians like Rapoport[3], Dresher[4],Raiffa[5], McKinsey[6] etc explained various situations of OR and useful applications of game theory.

II.BASIC FORMATION OF 10X10 GAME

The prime game is considered with 10 rows and 10 columns with increasing prime numbers according to player A and Player B. One player selects only one single action from his/her set possible actions. It consists of ten possible actions of A i.e A1,A2,A3,A4,A5,A6,A7,A8,A9,A10 which will effect on the other ten possible actions of player B i.e B1,B2,B3,B4,B5,B6,B7,B8,B9,B10. It is very much convenient to us to consider the influence of Player B on the components of Player A with maximum possible extent ten units. With this strategy, the game is constituted by increasing prime numbers in Actions of Player B with quantity ten units in actions of player A to meet the required criteria of row and column dominance in the game.

The pay-off matrix of the game having the size 10x10 is given below.

$$\begin{bmatrix} 2 & 3 & 5 & 7 & 11 & 13 & 17 & 19 & 23 & 29 \\ 31 & 37 & 41 & 43 & 47 & 53 & 59 & 61 & 67 & 71 \\ 73 & 79 & 83 & 89 & 97 & 101 & 103 & 107 & 109 & 113 \\ 127 & 131 & 137 & 139 & 149 & 151 & 157 & 163 & 167 & 173 \\ 179 & 181 & 191 & 193 & 197 & 199 & 211 & 223 & 227 & 229 \\ 233 & 239 & 241 & 251 & 257 & 263 & 269 & 271 & 277 & 281 \\ 283 & 293 & 307 & 311 & 313 & 317 & 331 & 337 & 347 & 349 \\ 353 & 359 & 367 & 373 & 379 & 383 & 389 & 397 & 401 & 409 \\ 419 & 421 & 431 & 433 & 439 & 443 & 449 & 457 & 461 & 463 \\ 467 & 479 & 487 & 491 & 499 & 503 & 509 & 521 & 523 & 541 \end{bmatrix}$$

III. MATERIAL AND METHODS

- The author applied Brown's algorithm to solve this special case of 10x10 game in which row and columns both dominated. Brown's Algorithm:
- Step 1: Player A chooses one of the possible actions(Ai_1) from A1-A10 to play, and Player B then plays with the possible action Bj_1 corresponding to the smallest element in the selected action Ai_1.
- Step 2: Player A then picks out the possible action (Ai_2) from A1 - A10 to play corresponding to the largest element in the possible action (Bj_1) selected by Player B in step 1.
- Step 3: Player B sums the actions of Player A who has played thus far, and plays with the possible action of Bj_2 corresponding to a smallest sum element.
- Step 4: Player A sums the actions of Player B who has played thus far, and plays the possible action (Ai_3) corresponding to a largest sum element. After the required iterations are computed,then go to step 5; otherwise, come back to step 3.
- Step 5: Compute an upper and lower bound $\underline{\gamma}$ and $\bar{\gamma}$ respectively.

$$\bar{\gamma} = \frac{\text{Largest sum element from step 4}}{\text{Number of plays of the game thus far}} \quad and \quad \underline{\gamma} = \frac{\text{Smallest sum element from step 3}}{\text{Number of plays of the game thus far}}$$

- Step 6:let Xi be the portion of the time Player A played row i with i=1,2,...,m and let Yi be the proportion of the time Player B played column j with j=1,2,...,n. These strategies approximate the optimal mini max strategies. Upper and Lower bounds of the value of the game where $\underline{\gamma} \leq \gamma \leq \bar{\gamma}$ are calculated in step 5. The Process completes.

IV. RESULTS

Brown's Algorithm is applied on prime problem up to 500 iterations to get good accuracy in the results with help of Java program. The influences of player B on each component of Player A are tabulated from Table (1) to Table (10) at each iteration.

Table-1: Player A vs Player B at 50th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
100	22885	22914	22956	23010	23062	23116	23166	23236	23302	23350
1550	23474	23508	23550	23602	23652	23710	23764	23830	23892	23950
3650	23868	23904	23946	24000	24054	24104	24170	24230	24294	24350
6350	24066	24102	24148	24198	24252	24310	24370	24432	24492	24550
8950	24462	24498	24548	24600	24648	24708	24764	24830	24890	24950
11650	24660	24700	24748	24798	24846	24910	24964	25030	25090	25150
14150	24958	25000	25044	25098	25152	25210	25272	25330	25390	25450
17650	25548	25590	25636	25692	25752	25800	25866	25926	25986	26050
20950	25650	25694	25736	25794	25854	25904	25974	26028	26088	26150
23350	26538	26580	26622	26682	26738	26790	26858	26918	26972	27050

Table-2: Player A vs Player B at 100th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
200	46235	46264	46306	46360	46412	46466	46516	46586	46652	46700
3100	47424	47458	47500	47552	47602	47660	47714	47780	47842	47900
7300	48218	48254	48296	48350	48404	48454	48520	48580	48644	48700
12700	48616	48652	48698	48748	48802	48860	48920	48982	49042	49100
17900	49412	49448	49498	49550	49598	49658	49714	49780	49840	49900
23300	49810	49850	49898	49948	49996	50060	50114	50180	50240	50300
28300	50408	50450	50494	50548	50602	50660	50722	50780	50840	50900
35300	51598	51640	51686	51742	51802	51850	51916	51976	52036	52100
41900	51800	51844	51886	51944	52004	52054	52124	52178	52238	52300
46700	53588	53630	53672	53732	53788	53840	53908	53968	54022	54100

Table-3: Player A vs Player B at 150th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
300	69585	69614	69656	69710	69762	69816	69866	69936	70002	70050
4650	71374	71408	71450	71502	71552	71610	71664	71730	71792	71850
10950	72568	72604	72646	72700	72754	72804	72870	72930	72994	73050
19050	73166	73202	73248	73298	73352	73410	73470	73532	73592	73650
26850	74362	74398	74448	74500	74548	74608	74664	74730	74790	74850
34950	74960	75000	75048	75098	75146	75210	75264	75330	75390	75450
42450	75858	75900	75944	75998	76052	76110	76172	76230	76290	76350
52950	77648	77690	77736	77792	77852	77900	77966	78026	78086	78150

62850	77950	77994	78036	78094	78154	78204	78274	78328	78388	78450
70050	80638	80680	80722	80782	80838	80890	80958	81018	81072	81150

Table-4: Player A vs Player B at 200th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
400	92935	92964	93006	93060	93112	93166	93216	93286	93352	93400
6200	95324	95358	95400	95452	95502	95560	95614	95680	95742	95800
14600	96918	96954	96996	97050	97104	97154	97220	97280	97344	97400
25400	97716	97752	97798	97848	97902	97960	98020	98082	98142	98200
35800	99312	99348	99398	99450	99498	99558	99614	99680	99740	99800
46600	100110	100150	100198	100248	100296	100360	100414	100480	100540	100600
56600	101308	101350	101394	101448	101502	101560	101622	101680	101740	101800
70600	103698	103740	103786	103842	103902	103950	104016	104076	104136	104200
83800	104100	104144	104186	104244	104304	104354	104424	104478	104538	104600
93400	107688	107730	107772	107832	107888	107940	108008	108068	108122	108200

Table-5: Player A vs Player B at 250 th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
500	116285	116314	116356	116410	116462	116516	116566	116636	116702	116750
7750	119274	119308	119350	119402	119452	119510	119564	119630	119692	119750
18250	121268	121304	121346	121400	121454	121504	121570	121630	121694	121750
31750	122266	122302	122348	122398	122452	122510	122570	122632	122692	122750
44750	124262	124298	124348	124400	124448	124508	124564	124630	124690	124750
58250	125260	125300	125348	125398	125446	125510	125564	125630	125690	125750
70750	126758	126800	126844	126898	126952	127010	127072	127130	127190	127250
88250	129748	129790	129836	129892	129952	130000	130066	130126	130186	130250
104750	130250	130294	130336	130394	130454	130504	130574	130628	130688	130750
116750	134738	134780	134822	134882	134938	134990	135058	135118	135172	135250

Table-6: Player A vs Player B at 300th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
600	139635	139664	139706	139760	139812	139866	139916	139986	140052	140100
9300	143224	143258	143300	143352	143402	143460	143514	143580	143642	143700
21900	145618	145654	145696	145750	145804	145854	145920	145980	146044	146100
38100	146816	146852	146898	146948	147002	147060	147120	147182	147242	147300
53700	149212	149248	149298	149350	149398	149458	149514	149580	149640	149700
69900	150410	150450	150498	150548	150596	150660	150714	150780	150840	150900
84900	152208	152250	152294	152348	152402	152460	152522	152580	152640	152700
105900	155798	155840	155886	155942	156002	156050	156116	156176	156236	156300
125700	156400	156444	156486	156544	156604	156654	156724	156778	156838	156900
140100	161788	161830	161872	161932	161988	162040	162108	162168	162222	162300

Table-7: Player A vs Player B at 350th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
700	162985	163014	163056	163110	163162	163216	163266	163336	163402	163450
10850	167174	167208	167250	167302	167352	167410	167464	167530	167592	167650
25550	169968	170004	170046	170100	170154	170204	170270	170330	170394	170450
44450	171366	171402	171448	171498	171552	171610	171670	171732	171792	171850
62650	174162	174198	174248	174300	174348	174408	174464	174530	174590	174650
81550	175560	175600	175648	175698	175746	175810	175864	175930	175990	176050
99050	177658	177700	177744	177798	177852	177910	177972	178030	178090	178150
123550	181848	181890	181936	181992	182052	182100	182166	182226	182286	182350
146650	182550	182594	182636	182694	182754	182804	182874	182928	182988	183050
163450	188838	188880	188922	188982	189038	189090	189158	189218	189272	189350

Table-8: Player A vs Player B at 400th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
800	186335	186364	186406	186460	186512	186566	186616	186686	186752	186800
12400	191124	191158	191200	191252	191302	191360	191414	191480	191542	191600
29200	194318	194354	194396	194450	194504	194554	194620	194680	194744	194800
50800	195916	195952	195998	196048	196102	196160	196220	196282	196342	196400
71600	199112	199148	199198	199250	199298	199358	199414	199480	199540	199600
93200	200710	200750	200798	200848	200896	200960	201014	201080	201140	201200
113200	203108	203150	203194	203248	203302	203360	203422	203480	203540	203600
141200	207898	207940	207986	208042	208102	208150	208216	208276	208336	208400
167600	208700	208744	208786	208844	208904	208954	209024	209078	209138	209200
186800	215888	215930	215972	216032	216088	216140	216208	216268	216322	216400

Table-9: Player A vs Player B at 450th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
900	209685	209714	209756	209810	209862	209916	209966	210036	210102	210150
13950	215074	215108	215150	215202	215252	215310	215364	215430	215492	215550
32850	218668	218704	218746	218800	218854	218904	218970	219030	219094	219150
57150	220466	220502	220548	220598	220652	220710	220770	220832	220892	220950
80550	224062	224098	224148	224200	224248	224308	224364	224430	224490	224550
104850	225860	225900	225948	225998	226046	226110	226164	226230	226290	226350
127350	228558	228600	228644	228698	228752	228810	228872	228930	228990	229050
158850	233948	233990	234036	234092	234152	234200	234266	234326	234386	234450
188550	234850	234894	234936	234994	235054	235104	235174	235228	235288	235350
210150	242938	242980	243022	243082	243138	243190	243258	243318	243372	243450

Table-10: Player A vs Player B at 500th Iteration

Player A					Player B					
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
1000	233035	233064	233106	233160	233212	233266	233316	233386	233452	233500
15500	239024	239058	239100	239152	239202	239260	239314	239380	239442	239500
36500	243018	243054	243096	243150	243204	243254	243320	243380	243444	243500
63500	245016	245052	245098	245148	245202	245260	245320	245382	245442	245500
89500	249012	249048	249098	249150	249198	249258	249314	249380	249440	249500
116500	251010	251050	251098	251148	251196	251260	251314	251380	251440	251500
141500	254008	254050	254094	254148	254202	254260	254322	254380	254440	254500
176500	259998	260040	260086	260142	260202	260250	260316	260376	260436	260500
209500	261000	261044	261086	261144	261204	261254	261324	261378	261438	261500
233500	269988	270030	270072	270132	270188	270240	270308	270368	270422	270500

4.1. Observation from the Iterations:

(i). Good correlations are observed in every iteration.

(ii).The influence of Player B uniformly effects on the possible action of PlayerA in each iteration.

(iii).Nominal variations only are identified at each the iteration.

(iv).Disturbed Fluctuations are not observed.

(v).Constant differences between the values of possible actions of player A at any two consequent iterations have been determined

(vi).Constant differences between the values of possible actions of player B at any two consequent iterations have been ascertained.

V. OPTIMUM MIXED STRATEGIES OF PLAYER A AND PLAYER B:

The optimum mixed strategies of Player A and Player B are shown in **Table-11.**

Table-11

Opitimum Mixted strategies of Player A and Player B (Iteration wise)																			
50		100		150		200		250		300		350		400		450		500	
A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

VI. LOWER BOUNDS AND UPPER BOUNDS AT ALL ITERATIONS:

At each play of the game the minimum sum element selected by player B divided by the number of place of the game is known as lower bound.

Similarly At each play of the game the maximum sum element selected by player A divided by the number of place of the game is called as upper bound.

The Values of U.Bs and L.Bs in 10 x 10 game are tabulated in Table-12.

Table-12

U.B	Lower Bounds									
Iterations 50-500	50	100	150	200	250	300	350	400	450	500
467	457.7	462.35	463.90	464.67	465.14	465.45	465.67	465.83	465.96	466.07
467	458.28	462.64	464.09	464.82	465.25	465.54	465.75	465.91	466.03	466.12
467	459.12	463.06	464.37	465.03	465.42	465.68	465.87	466.01	466.12	466.21
467	460.20	463.60	464.73	465.30	465.64	465.86	466.02	466.15	466.24	466.32
467	461.24	464.12	465.08	465.56	465.84	466.04	466.17	466.28	466.36	466.42
467	462.32	464.66	465.44	465.83	466.06	466.22	466.33	466.41	466.48	466.53
467	463.32	465.16	465.77	466.08	466.26	466.38	466.47	466.54	466.59	466.63
467	464.72	465.86	466.24	466.43	466.54	466.62	466.67	466.71	466.74	466.77
467	466.04	466.52	466.68	466.76	466.80	466.84	466.86	466.88	466.89	466.90
467	467	467	467	467	467	467	467	467	467	467

VII. CONCLUSIONS

(i).The value of the prime game is 467.

(ii).The optimum mixed strategies of player A and player B are unique at each iteration.

(iii).The same upper bound value is obtained at any stage of iteration.

(iv).The Initial error is 9.3.It will be minimized later.

(v).The best strategies are obtained for Player A & Player B.

(vi).The considered game is classified as a strictly determinable game because of the same values of lower bound and upper bound at final stage.

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