EVIDENTIAL REASONING APPROACH TO ANALYZE VALIDITY OF REPORTS BY HISTORIANS ON MAHMUD OF GHAZNI'S INVASION (1001AD-1027AD)

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ABSTRACT

There is an innate desire within us to know the truth. We assign the tag 'TRUE' to a conclusion or a final decision if the evidences are genuine and the inference is logically valid. With regard to historical writings even among reputed historians we find conflicting statements and conclusions. It could have happened due to reasons like false inputs, subjective interpretations and ambiguous comments. Invasion of Mahmud of Ghazni had been reported by many historians. For our analysis, we have taken seven famous historians. Using the concepts of Multi Criteria Decision Making methods and its refined tool "Evidential Reasoning approach", we analyze the reporting by seven historians on the Invasion of Mahmud of Ghazni starting from 1001 AD onwards.

Keywords: Evidential Reasoning Approach (ER), Historians, India, Mahmud Of Ghazni, Multiple-Criteria Decision-Analysis (MCDA).

I. INTRODUCTION

An incident is reported in the media today and in future it becomes history. With all the technological developments, we aware of the biased or even paid reporting. It is our human nature or even there is an innate longing within us to know the truth. When we read the history we believe that the past events had been reported sincerely and truthfully. When we get conflicting reports even by reputed historians on the same incident, we make use of our logical ability to infer the truth.

Deductive reasoning or laws of inference are the existing logical tools to filter truth from false information. As an outcome of objective reasoning methodologies, we arrive at a conclusion or a decision on the issues discussed. The final decision becomes a judgment on the issues. Thus inferring the truth and making a judgment on the event or issues could be considered as a decision making process.

In the field of management studies, Decision making involves projections about the outcome or probabilities that could occur in future; whereas decision making on the conflicting reports of the past events, it depends on the hypothesis formed by the historians. As an ardent learner, we do not have access to primary sources on the reports of past events but at most we may get access to the secondary sources. As a sincere lover of truth, we

limit our analysis to the historical reports published in book format rather than the primary or secondary sources. Using fuzzy tools, our objective is to make sense out of the conflicting historical reports.

Often we find ourselves in a situation where we cannot fully agree or absolutely deny the truth value of a historical reporting. Contradictory or conflicting reports leave the reader in a confusing state or technically 'uncertain mode'. Fuzzy logic approach is the right way to handle uncertain information. In 500 BC, Buddha based his teachings on the fact that the world is filled with contradictions, almost everything in the world has its opposite; something exists and it doesn't exist at the same time. Buddha's philosophy has connection to modern fuzzy logic. The fuzzy logic has reached a new milestone in the year of 1965 when Prof. Zadeh. L. A. introduced *fuzzy sets*[1].

Prof. Zadeh. L. A. gave the theoretical explanation and validity for the fuzzy sets and their remarkable functions. Bellman. R. E. and Zadeh. L. A. (1970) followed by Ronald R. Yager and Thomas L. Saaty in the last few decades of 20th century, contributed to fuzzy logic applications. Improvements in Fuzzy Multiple Criteria Decision Making (MCDM) model was proposed by Ronald R. Yager. And decision-making frame work was used for extensive Multiple-criteria decision-analysis (MCDA) by Thomas L. Saaty and also he proposed Analytic Hierarchy Process (AHP) method[2].

There are many decision making methodologies available in the research area like Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Technique for the Order of Preference by Similarity to Ideal Solution (TOPSIS), VIKOR, Multi Attribute Value Theory (MAVT), Multi Attribute Utility Theory (MAUT), PROMETHEE (Outranking), ELECTRE (Outranking), Evidential Reasoning approach (ER) and so on. We employ the tool proposed by Yang and Xu in 2000, namely, Evidential Reasoning approach. When multiple information are available we employ Multiple Criteria Decision Making methods and ER method bases it basic principles on MCDM. It also incorporates our belief that a particular historian's reporting is TRUE and FACTUAL. ER incorporates beliefs and disbeliefs.

When a student repeatedly attempts to clear backlog of arrears or when someone tries again and again to succeed, often the elders give the example of Mahmud of Ghazni who invaded Indian territories during the period 1001 AD. Different historians like A. L. Basham, Dr. B.R. Ambedkar, W.W. Hunter, Stanley Lane-Poole, Romila Thapar, Sidney Owen and Mohammad Habib have reported the event.

In this work we attempt to analyze the objectives enunciated by the famous historians on the same event of invasion of Mahmud of Ghazni in 1001 AD.

The significance of the Evidential Reasoning approach is present in extended decision Matrix comparing to other MCDA. In which, each attribute of an alternative is described by a distributed assessment using belief degree. Evidential reasoning is a general evidence-based Multi-criteria decision-analysis (MCDA) approach for dealing problems under various uncertainties including ignorance and randomness.

We use the Evidential Reasoning approach to analyze the derivation made by the historians. Based on the result of our Evidential Reasoning approach we make our conclusions or judgment on the objectives hypothecated by the historians on the same event "Invasion of Mahmud of Ghazni in 1001 AD". In the succeeding section, we enumerate the literature survey on Evidential Reasoning approach. The sections Three describe the Evidential Reasoning approach's frame work with its algorithm. The historical event Invasion of Mahmud of Ghazni on

India has been discussed as a case study in chapter four. As per to ER analysis, the conclusion is drawn by ranking the historical reports which are closer to the reality.

II. LITERATURE SURVEY

The Evidential Reasoning (ER) approach is a common approach for analyzing Multiple Criteria Decision Making (MCDM) problems under uncertainties. The Evidential Reasoning approach was developed on the basis of Dempster-Shafer evidence theory[3] and decision theory. The concepts of belief structure were introduced by Yang and Xu[4]; Yang and Singh[5] and Zhang et al.[6]. Also the belief decision matrix was developed by Xu and Yang[7] initially which made easier to model uncertainties of various types of nature in a unified format for further analysis without resorting to sensitivity analysis. The extension of the modeling technique to model other types of uncertainty, such as interval uncertainties was introduced by Fan and Deer and Xu et al.[8] and also fuzzy uncertainties was shown by Yang et al.[9] and uncertainties in other parameters of a decision problem such as criterion weights and belief degrees by Guo et al.[10].

The use of the belief structure to extend traditional rule based expert systems to belief rule based expert systems which not only allows uncertainties to be explicitly modeled in rules, but also equips a belief rule based system with the capability to learn and model complex casual relationships by Yang et al.[11] and Xu et al.[12]. Incorporation of the Evidential Reasoning algorithm in various decision support tools, such as Intelligent Decision System (IDS) by Xu and Yang, were developed including a web based assessment tool by Xu and groupware by Iourinski and Ramalingam. And the comparison studies of the approach were made with other approaches such as AHP by Thomas L. Saaty and neural networks by Xu and Yang; Wang and Elhag[13].

III. EVIDENTIAL REASONING APPROACH: FRAME WORK AND ALGORITHM

3.1 Evidential Reasoning Algorithm

3.1.1 Evaluation Analysis Model (Problem Description)

The Evidential Reasoning approach has to deal with both quantitative and qualitative attributes. The Subjective judgments may be used to differentiate one alternative from another on qualitative attributes.

To evaluate the ability of an authors' view on historical event, for example, the subjective judgments may be that "an author's view on historical event is *false*, *true*, or *exactly true* to certain degrees." In such judgments, *false*, *true*, and *exactly true* denote distinctive evaluation grades. For this evaluation problem, a set of evaluation grades is define by, $H = \{Absolutely\ false(H_1)\ False(H_2)\ Not\ Infer(H_3)\ True(H_4)\ Exactly\ true(H_1)\}$ *Author1's* view of historical event is a general and difficult to assess directly. It needs to be decomposed into detailed based on some criteria, such as *statement 1*, *statement 2* and *statement 3*. If a detailed concept is still too abstract to assess directly, it may be further broken down to more detailed concepts. For instant, the concept (attribute) of *Author1* $y(e_1)$ may be measured by *statement 1* (e_{11}), *statement 2* (e_{12}) and *statement 3* (e_{13}) which can be directly assess based on criteria. An evaluation hierarchy for assessing this problem is shown in Fig. 1.

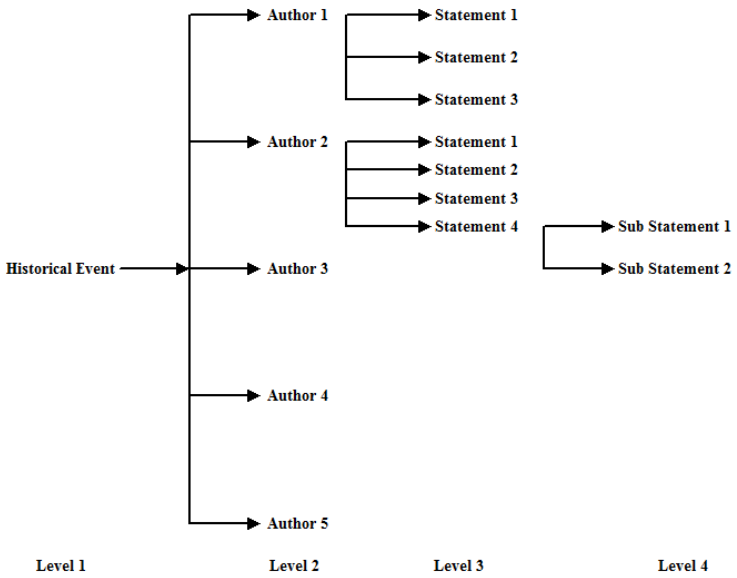

Figure 1: Evaluation Hierarchy for Assessing Author's view for Historical Event

In hierarchical assessment, a high level attribute is assessed through associated lower level attributes. For example, if the *statement 1*, *statement 2* and *statement 3* of *Historical event* are all assessed to be accurately *true*, then the *author 1's view* should also be *true*. In evaluation of qualitative attributes, uncertain judgments could be used. In assessment of the *author 1's view* of the *historical event*, for example, assessment may be

1) 30% sure that the *statement 1* is *Not infer* and 60% sure that it is *True*;
2) absolutely sure that the *statement 2* is *True*;
3) 50% sure that the *statement 3* is *True* and 50% sure that it is *Exactly true*.

In the above assessments, 30%, 50%, 60%, and 100% (absolutely sure) are referred to as degrees of belief and sometimes used in decimal format: 0.3, 0.5, 0.6, and 1, respectively. Note that assessment 1) is incomplete as the total degree of belief is 0.3 + 0.6 = 0.9 < 1 while assessments 2) and 3) are complete. The missing 0.1 in assessment 1) represents the degree of ignorance or uncertainty. A problem arises as to how to generate an overall assessment about the *author 1*'s view of *historical event* by aggregating the above three judgments in a rational way. The Evidential Reasoning approach provides a means for dealing with the aggregation problem. The basic Evidential Reasoning model and the ER algorithm are discussed in the following subsections, and the synthesis axioms are defined in the following subsection where the shortcomings of the original ER algorithm are also discussed.

3.1.2 Basic Evaluation Framework

To begin with, suppose there is a simple two-level hierarchy of attributes with a general attribute at the top level and a number of basic attributes at the bottom level. Suppose there are k basic attributes $e_i (i = 1, 2, \dots k)$ associated with a general attribute y. Define a set of k basic attributes as follows: $E = \{e_1\ e_2\ e_3\ \dots\ e_i\ \dots\ e_k\}$ ……(1) Suppose the weights of the attributes are given by $w = \{w_1\ w_2\ w_3\ \dots\ w_i\ \dots\ w_k\}$ where w_i is the relative weight of the ith basic attribute (e_i) with $0 \leq w_i \leq 1$. Weights play an important role in assessment. They may be estimated using existing methods such as simple rating methods or more elaborate methods based on the pair wise comparisons of attributes.

Suppose N distinctive evaluation grades are defined that collectively provide a complete set of standards for assessing an attribute, as represented by $H = \{H_1\ H_2\ H_3 \dots H_n \dots H_N\}$ .....(2), where H_n is the nth evaluation grade. Without loss of generality, it is assumed that H_{n+1} is preferred to H_n.

A given assessment for $e_i (i = 1, 2, \dots k)$ of an alternative may be mathematically represented as the following distribution: $S(e_i) = \{(H_n, \beta_{n,i}), n = 1, \dots, N\} \quad i = 1, 2, \dots, k$ .....(3), where $\beta_{n,i} \geq 0$, $\sum_{n=1}^{N} \beta_{n,i} \leq 1$, and $\beta_{n,i}$ denotes a degree of belief. The above distributed assessment reads that the attribute e_i is assessed to the grade H_n with the degree of belief of $\beta_{n,i}$, $n = 1, \dots, N$. An assessment $S(e_i)$ is complete if $\sum_{n=1}^{N} \beta_{n,i} = 1$ and incomplete if $\sum_{n=1}^{N} \beta_{n,i} < 1$. A special case is $\sum_{n=1}^{N} \beta_{n,i} = 0$ (or $\beta_{n,i} = 0$ for all $n = 1, \dots, N$), which denotes a complete lack of information on e_i. Such partial or complete ignorance is not rare in many decision making problems. In the new ER approach to be investigated in this and next sections, ignorance will be handled using the upper and lower bounds of degrees of belief and utility intervals.

Let β_n be a degree of belief to which the general attribute y is assessed to the grade H_n. The aggregation problem is to generate $\beta_n (n = 1, \dots, N)$ by aggregating the assessments for all the associated basic attributes $e_i (i = 1, 2, \dots k)$ as given in equation (3). The following Evidential Reasoning algorithm can be used for this purpose.

3.1.3 New Evidential Reasoning Algorithm

The new Evidential Reasoning algorithm, decomposed into two parts as $\bar{m}_{H,i}$ and $\tilde{m}_{H,i}$, where $\bar{m}_{H,i} = 1 - w_i$ and $\tilde{m}_{H,i} = w_i\left(1 - \sum_{n=1}^{N} \beta_{n,i}\right)$ ..... (4) with $m_{H,i} = \bar{m}_{H,i} + \tilde{m}_{H,i}$. $\bar{m}_{H,i}$ is the first part of the remaining probability mass that is not yet assigned to individual grades due to the fact that attribute i (denoted by e_i) only plays one part in the assessment relative to its weight. $\bar{m}_{H,i}$ is a linear decreasing function of w_i. $\bar{m}_{H,i}$ will be one if the weight of e_i is zero or $w_i = 0$; $\bar{m}_{H,i}$ will be zero if e_i dominates the assessment or $w_i = 1$. In other words, $\bar{m}_{H,i}$ represents the degree to which other attributes can play a role in the assessment. $\bar{m}_{H,i}$ should eventually be assigned to individual grades in a way that is dependent upon how all attributes are weighted and assessed.

$\tilde{m}_{H,i}$ is the second part of the remaining probability mass unassigned to individual grades, which is caused due to the incompleteness in the assessment $S(e_i)$. $\tilde{m}_{H,i}$ will be zero if $S(e_i)$ is complete, or $\sum_{n=1}^{N} \beta_{n,i} = 1$; otherwise, $\tilde{m}_{H,i}$ will be positive. $\tilde{m}_{H,i}$ is proportional to w_i and will cause the subsequent assessments to be incomplete.

The following new Evidential Reasoning algorithm is then developed for combining the first 'i' assessments with the $(i+1)^{th}$ assessment using the same process as shown in *Table 4.1* in a recursive manner

$$\{H_n\}: m_{n,I(i+1)} = K_{I(i+1)}\left[m_{n,I(i)} m_{n,i+1} + m_{H,I(i)} m_{n,i+1} + m_{n,I(i)} m_{H,i+1}\right] \dots\dots (5a)$$

$$m_{H,I(i)} = \tilde{m}_{H,I(i)} + \bar{m}_{H,I(i)} (n = 1, 2, \dots, N) \dots\dots (5b)$$

$$\{H\}: \tilde{m}_{n,I(i+1)} = K_{I(i+1)}\left[\tilde{m}_{H,I(i)} \tilde{m}_{H,i+1} + \bar{m}_{H,I(i)} \tilde{m}_{H,i+1} + \tilde{m}_{H,I(i)} \bar{m}_{H,i+1}\right] \dots\dots (5c)$$

$$\{H\}: \bar{m}_{H,I(i+1)} = K_{I(i+1)}\left[\bar{m}_{H,I(i)} \bar{m}_{H,i+1}\right] \dots\dots (5d)$$

$$K_{I(i+1)} = \left[1 - \sum_{t=1}^{N} \sum_{\substack{j=1 \\ j \neq t}}^{N} m_{t,I(i)}\, m_{j,i+1}\right]^{-1} \qquad i = \{1, 2, \dots, k-1\} \;\dots\dots(5e)$$

The terms $\bar{m}_{H,I(i)}\,\tilde{m}_{H,i+1}$ and $\tilde{m}_{H,I(i)}\,\bar{m}_{H,i+1}$ are assigned to $\tilde{m}_{H,I(i+1)}$ rather than to $\bar{m}_{H,(i+1)}$ so that the incompleteness synthesis axiom can be satisfied. After all k assessments have been aggregated, the combined degrees of belief are generated by assigning $\bar{m}_{H,I(k)}$ back to all individual grades proportionally using the following normalization process: $\{H_n\}: \beta_n = \frac{m_{n,I(k)}}{1-\bar{m}_{H,I(k)}} \quad n = 1, 2, \dots, N \;\dots\dots(6a)$

$$\{H\}: \beta_H = \frac{\tilde{m}_{H,I(k)}}{1-\bar{m}_{H,I(k)}} \;\dots\dots\; (6b)$$

β_n generated above is a likelihood to which H_n is assessed. β_H is the unassigned degree of belief representing the extent of incompleteness in the overall assessment. In summary equation $(5a) - (5e)$ for attribute aggregation. Also equations $(6a)$ and $(6b)$ for generating combined degrees of belief. Similar to equation (3), the generated assessment for y can be represented by the following distribution: $S(y) = \{(H_n, \beta_n), n = 1, 2, \dots, N\} \;\dots\dots\; (7)$, which reads that y is assessed to the grade H_n with the degree of belief of $\beta_n (n = 1, 2, \dots, N)$.

3.1.4 Expected Utility and Utility Interval of the Evidential Reasoning Approach

The concept of expected utility is used to define such values. Suppose $u(H_n)$ is the utility of the grade H_n with $u(H_{n+1}) > u(H_n)$ if H_{n+1} is preferred to H_n. $u(H_n)$ may be estimated using the probability assignment method or by constructing regression models using partial rankings or pairwise comparisons. If all assessments are complete and precise, there will be $\beta_H = 0$ and the expected utility of the attribute y can be used for ranking alternatives, which is calculated by $u(y) = \sum_{n=1}^{N} \beta_n u(H_n) \;\dots\dots\dots\; (8)$. An alternative a is preferred to another alternative b on y if and only if $u(y(a)) > u(y(b))$.

If any assessment for the basic attribute is incomplete, it will be proven that β_H is positive. Within the Evidential Reasoning assessment framework, β_n given in $(6a)$ represents the belief measure in the Dempster–Shafer theory and thus provides the lower bound of the likelihood to which y is assessed to H_n. The upper bound of the likelihood is given by a plausibility measure. It can be shown that the plausibility measure for H_n within the Evidential Reasoning evaluation framework is given by $(\beta_n + \beta_H)$. Thus the belief interval $[\beta_n, (\beta_n + \beta_H)]$ provides the range of the likelihood to which y may be assessed to H_n. It is obvious that the interval will reduce to a point β_n if all assessments are complete.

The above discussion shows that if any basic assessment is incomplete, the likelihood to which y may be assessed to H_n is not unique and can be anything in the interval $[\beta_n, (\beta_n + \beta_H)]$. In such circumstances, we define three measures to characterize the assessment for y, namely the minimum, maximum and average expected utilities. Without loss of generality, suppose H_1 is the least preferred grade having the lowest utility and H_N the most preferred grade having the highest utility. Then the maximum, minimum andaverage expected utilities on are given by $u_{max}(y) = \sum_{n=1}^{N-1} \beta_n u(H_n) + (\beta_N + \beta_H) u(H_N) \;\dots\; (9)$, $u_{min}(y) = (\beta_1 + \beta_H) u(H_1) + \sum_{n=2}^{N} \beta_n u(H_n) \;\dots\; (10)$, $u_{avg}(y) = \frac{u_{max}(y) + u_{min}(y)}{2} \;\dots\; (11)$, If all original

assessments $S(e_i)$ are complete, then $\beta_H = 0$ and $u(y) = u_{max}(y) = u_{min}(y) = u_{avg}(y)$. Note that the above utilities are only used for characterizing an assessment but not for attribute aggregation.

The ranking of two alternatives a_l and a_k is based on their utility intervals. a_l is said to be preferred to a_k on y if and only if $u_{min}(y(a_l)) > u_{max}(y(a_k))$; a_l is said to be indifferent to a_k if and only if $u_{min}(y(a_l)) = u_{min}(y(a_k))$ and $u_{max}(a_l) = u_{max}(a_k)$. Otherwise, average expected utility may be used to generate a ranking, though such a ranking is inconclusive.

For instance, if $u_{avg}(y(a_l)) > u_{avg}(y(a_k))$ but $u_{min}(y(a_l)) < u_{max}(y(a_k))$, one could say that a_l is preferred to a_k on an average basis. However, this ranking is not reliable, as there is a chance that a_k may have higher utility than a_l. In such cases, to generate a reliable ranking the quality of the original assessments must be improved by reducing incompleteness present in the original assessments associated with a_l and a_k.

IV. EVIDENTIAL REASONING APPROACH TO THE WRITINGS ON MAHMUD OF GHAZNI'S INVASION (1001 AD – 1027AD)

The historical event which is considered in this case study is the "Invasion of Mahmud of Ghazni on India (1001 AD – 1027 AD)". Particularly, Mahmud's sixteenth expedition was on temple of Somnath. On this event Romila Thapar, a researcher as well as a historian says, "*The conversion of the event into a symbol in the politics of nationalism in India in the twentieth century began in western India. Somanatha was said to be symbolic of Hindu subjugation and the ensuing trauma over Muslim rule. The reconstruction of the temple at Somanatha was demanded. This became a contentious issue between, what are now described as secular nationalists, and those with an agenda that perceived politics in terms of religious identities. This tension was brought to a head by the rebuilding of the temple at Somanatha in 1951".*And each narrator has written or portrayed Mahmud's raid on India in different perspective. "*There are many concerns that weave their way through this analysis of an event: the subsequent history, the historiography and the interconnection may illumine our understanding of the event… The first two aspects focus on what happened and how it has been interpreted, and the third concentrates on the point in time and space when memory is introduced into the interpretation. Each narrative is connected to the history of the place, but each narrative is also connected to what it perceives as the politics of power. The narratives, therefore, are at times ambiguous and more frequently conflicting… The perception of Mahmud at the popular level in a largely oral tradition;… and the Indian nationalist reconstruction of the event."[14]*

For the case study analysis, the writing of few authors on "Invasion of Mahmud of Ghazni on India" is taken up the authors are, 1) Basham. A. L., 2) Ambedkar. B. R., 3) Hunter. W. W., 4) Stanley Lane-Poole, 5) Romila Thapar, 6) Sidney Owen and 7) Mohammad Habib.

4.1 Evaluation on Case Study

4.1.1 Basic Evaluation Framework

Invasion of 'Mahmud of Ghazni' on India: this problem is to evaluate the state of attributes y_k and sub attributes or factors $y_{k,i}$ at alternatives a_r within the Evidential Reasoning framework. Since the attributes y_k are the

authors, who have written the "Invasion of Mahmud of Ghazni" and the sub attributes $y_{k,i}$ are writings on "Invasion of Mahmud of Ghazni". The most conflicting information regarding the purpose of 'Mahmud's raid on India' has been considered as alternatives a_γ which has been interpreted by various aspects. We first define the set of evaluation grades for y_k as, $H = \{H_1 \;\; H_2 \;\; H_3 \;\; H_4 \;\; H_5 \;\; H_6 \;\; H_7 \;\; H_8 \;\; H_9\}$

$H_1 \rightarrow$ Extremely False; $H_2 \rightarrow$ Strongly False; $H_3 \rightarrow$ False; $H_4 \rightarrow$ Moderately False; $H_5 \rightarrow$ Not Infer; $H_6 \rightarrow$ Moderately True; $H_7 \rightarrow$ True; $H_8 \rightarrow$ Strongly True; $H_9 \rightarrow$ Extremely True and the evaluation grades are scaled as $(-1,\ 1)$, $H = \{-1,\ -0.75,\ -0.5,\ -0.25,\ 0,\ 0.25,\ 0.5,\ 0.75,\ 1\}$ where $P(H) = \sum_{n=1}^{N} \frac{P(H_n)}{N} = 0$; $H_n (n = 1, 2, \ldots, N)$ are supposed to be distinct grades.

4.1.2 Attributes, Factors and its Weights

Let the attributes be y_k, $k = 1, 2, \ldots, 7$ and sub attributes (factors) be $y_{k,i}$. The weight of each attributes is denoted by w, where $w = (w_k)(k = 1, 2, \ldots, 7)$ are scaled between 1 and 9. Similarly the sub attributes are denoted by $w_{k,i}$, which are equal according to the attributes. These are as follows:

$y_1 \rightarrow$ A. L. Basham[15]; $y_{1,1} \rightarrow$ …From 1001 A.D. Mahmud made seventeen great raids on India. The whole western half of the land felt the force of the Turuskas; palace and temples were looted and desecrated and enormous caravans of booty and slaves were taken back to Ghazni.

$y_{1,2} \rightarrow$ The raids reached as far as the great shrine of Somnath in Saurashtra and the kingdom of the Candellas in Bundelkhand. Among India's great cities Kanyakubja and Mathura were captured and plundered.

$y_{1,3} \rightarrow$ Mahmud did not remain in India, however, for, though Muslim chroniclers depict him as a staunch propagator of Islam, intent on converting the infidel and bringing India under the control of the true faith, his expeditions were for the purpose rather of plunder than of conquest.

Since, the author A. L. Basham is a well known western historian and he has referred by the history researchers and historians consider his weightage as $\hat{w}_1 = 9$.

$y_2 \rightarrow$ Dr. B.R. Ambedkar[16]; $y_{2,1} \rightarrow$ …Soon after this withdrawal, there began a series of terrible invasions by Muhammad of Ghazni in 1001 A.D. Muhammad died in 1030 A.D., but within the short span of 30 years, he invaded India 17 times… For thirty years had Muhammad of Ghazni ravaged India and for thirty years Mahommad Ghori harried the same country in the same way…

$y_{2,2} \rightarrow$ These invasions of India by Muslims were as much wars among the Muslims themselves. This fact has remained hidden because the invaders are all lumped together as Muslim without distinction… They were not a loving family cemented by the feeling of Islamic brotherhood. They were deadly rivals of one another and their wars were often wars of mutual extermination.

$y_{2,3} \rightarrow$ What is, however, important to bear in mind is that with all their internecine conflicts they were all united by one common objective and that was to destroy the Hindu faith.

And the author Dr. B. R. Ambedkar is a well known scholar, his findings are close to the reality; and based on evidence. Also he has been referred by many researchers. Therefore we consider his weightage as $\hat{w}_2 = 9$.

$y_3 \rightarrow$ W.W. Hunter[17]; $y_{3,1} \rightarrow$ But each expedition ended by further strengthening the Muhammadan foothold in India. Mahmud carried away enormous booty from the Hindu temples, such as Thaneswar and Nagarkot, and

his sixteenth and most famous expedition was directed against the temple of Somnath in Gujarath (1024 A.D). After bloody repulses, he stormed the town; and the Hindu garrison, leaving 5000 dead, put out in boats to sea. The famous idol of Somnath was merely one of the twelve lingas or phallic emblems erected in various parts of India. But Mahmud having taken the name of the 'Idol-Smasher,' the modern Persian historians gradually converted the plunder of Somnath into a legend of his pious zeal. Forgetting the contemporary accounts of the idol as a rude stump of stone. But Mahmud cried out that he would rather be remembered as the breaker than the seller of idols, and clove the god open with his mace. Forthwith a vast treasure of jewels poured forth from its vitals, which explained the liberal offers of the priests, and rewarded the disinterested piety of the monarch. The growth of this myth can be clearly traced, but it is still repeated by uncritical historians. The linga or sohd stone fetish of Somnath, had no stomach, and could contain no jewels.

$y_{3,2} \rightarrow$ Mahmud carried off the temple gates, with fragments of the phallic emblem, to Ghazni, and on the way nearly perished with his army in the Indus desert.

$y_{3,3} \rightarrow$ As the result of seventeen invasions of India, and twenty-five years' fighting, Mahmud had reduced the western districts of the Punjab to the control of Ghazni, and left the remembrance of his raids as far as Kanauj on the east, and Gujarat in the south. He never set up as a resident sovereign in India. His expeditions beyond the Punjab were the adventures of a religious knight-errant with the plunder of a temple city, or the demolition of an idol, as their object, rather than serious efforts at conquest… The Muhammadan chroniclers tell many stories, not only of Mahmud's valour and piety, but also of his thrift.

The author W. W. Hunter is a western historian and he wrote this from western perspective. Therefore we consider his weightage as $\hat{w}_3 = 7$.

$y_4 \rightarrow$ Stanley Lane-Poole[18]; $y_{4,1} \rightarrow$ …when we speak of the Mohammedan empire in India we mean the rule of the Turks. Their invasion was no part of the expansion of Islam as a religious movement. It was merely the overflow of the teeming cradle-land of central Asia, …

$y_{4,2} \rightarrow$ Jaipal and fifteen of his kindred were brought captives before the conqueror. Their jeweled necklaces, worth, it is said, ninety thousand guineas apiece, were torn off, and half a million of slaves, and booty past counting, according to the florid statements of the oriental historians, fell into the hand of the Muslims. Mahmud was not cruel; he seldom indulged in wanton slaughter; …

$y_{4,3} \rightarrow$ …Year after year Mahmud swept over the plain of Hindustan, capturing cities and castles, throwing down temples and idols, and earning his title of 'Victor' and Idol-breaker,' Ghazi and Batshikan…

Since, the author Stanley Lane-Poole is a well known western historian and he is referred by many. We considered his weightage as $\hat{w}_4 = 8$.

$y_5 \rightarrow$ Romila Thapar[19]; $y_{5,1} \rightarrow$ Mahmud's greed for gold was insatiable, so his raids were directed to major temple towns such as Mathur, Thaesar, Kanauj and finally Somanatha. The concentration of wealth at Somanadha was renowned, so it was inevitable that Mahmud would have attacked it. Added to the desire for wealth was the religious motivation, iconoclasm being a meritorious activity among some followers of Islam. Somantha had a large income from the taxes paid by pilgrims who visited the temple, money that was sometimes forcibly appropriated by unscrupulous local rajas, according to local inscriptions. Attempts to prevent this were a major headache to the Chaulukya administration. Arab sources refer to temples making

profits on commercial investments, and Somanatha adjoined the commercially active port of veraval. The most profitable item in this trade was the import of horses that enriched both those who imported them and those who bought them for further distribution to the hinterland. An additional reason for Mahmud's determination to attack Somanatha may have been to reduce the import of horses from Arab traders. This would have benefited the traders of Ghazni who imported horses into north-west India, a trade mentioned in inscriptional sources.

$y_{5,2}$ → In 1026 A. D. Mahmud raided Somanatha, desecrated the temple and broke the idol. The event is described in Turko-Persian and Arab sources, some contemporary – the author claiming to have accompanied Mahmud- and others of later times, the story being repeated continually in these histories up to the seventeenth century… But there is no unanimity about the idol in other accounts. The earlier descriptions of the event identify it with the idol of Manat, a Pre-Islamic goddess of southern Arabia, whose shrine the prophet Mahmud had wanted destroyed and the idol broken; others write that it was a lingam; still others state that it was an anthropomorphic figure stuffed with jewels. Gradually a mythology was constructed around the temple and their idol, with alternative narratives.

$y_{5,3}$ → There is mach fantasy in such accounts and they have to be seen in the historiographical context of the gradual change in the projection of Mahmud from an iconoclast and plunder to the founder of Islamic rule in India even if the later is not quite what he was. The historiography of the raid on Somanatha has its own history. The popular view is that Mahmud's raid on Somanatha was such a trauma for the Hindus that it became seminal to the Hindu-Muslim antagonism of recent times. Yet there is no reference in contemporary or near contemporary local sources of the raid on Somanatha, barring a passing mention in a Jaina text, nor is there any discussion of what might have been a reaction, let alone a trauma among Hindus.

$y_{5,4}$ → Mahmud's iconoclasm earned him a title from the Caliph of Beghdad and recognition as a champion of Islam. Alberuni's comment on Mahmud's raids was that they caused economic devastation in the area, quite apart from the looting of temples. Nevertheless, judging by the evidence of the history of Somanatha and its vicinity subsequent to the raid, there was an impressive bouncing back of the local authority and of the economy. Given the frequency of various campaigns, some degree of periodic destabilization was probably a familiar experience of these times.

And the author Romila Thapar a well known historian, scholar and her research findings are close to the historical evidence. Also she is referred by the researchers and pupil so we consider her weightage as $\hat{w}_5 = 9$.

y_6 → Sidney Owen[20]; $y_{6,1}$ → In 930 A. H. (1000 A. D.) Mahmud left for India from Ghazni and conquered many forts… Soon the battle began. God gave victory to the Muslims. Mahmud was victorious. Jaipal was defeated… A good deal of booty was taken… From here Mahmud left for India. In 399 A. H. (1008-1009 A. D.) he fought against Anand pal and defeated him… Mahmud then succeeded in entering the fort with some of his companions. Gold, Silver and diamonds that had been accumulating, since the days of Bhim Pando, in this fort fell into his hands. Booty beyond counting fell into Mahmud's hands… A throne of gold and silver was built. The booty was displayed at Ghazni for the people to stare at.

$y_{6,2}$ → In 402 A. H. (1012 A. D.), Mahmud left Ghazni for Thanesar… When Mahmud reached Thanesar, he found the city deserted. Whatever fell into the hands of his man was destroyed. Many of idols were broken. Jogar Om (which was the famous idols in that Mecca of the Hindus) was carried away to Ghazni and placed at

the Durgah. People flocked to see it. In the year 404 A. H. (1014 A. D.), Mahmud decided to take Nanda… Mahmud, with some of his men, entered the fort and took away all the valuables and arms that were in the fort… Mahmud so arranged matters that the forts in the pass were taken and pillaged. His army captured a good deal of property and a large numbers of men. Many Hindus accepted Islam. The same year he issued orders that in the place conquered, mosques be raised and Hindus be converted to Islam by men appointed for the purpose.

$y_{6,3}$ → From here Mahmud advanced to Mahaban near Muttra on the Jumna then under Kala Chand… Mahaban was taken, 165 elephants and booty beyond imagination fell into Mahmud's hands. From here Mahmud advanced on Muttra, a very great city of the Hindus, Sacred as the birth place of Krishna son of Vasudeva. Here is a great Hindu temple. When Mahmud reached Muttra no one opposed him. He ordered his men to spread over the whole kingdom, destroy all idols or burn them and take possession of all property. From the temples, treasures and property beyond counting fell into Mahmud's hands. One sapphire weighed 450 mithgals. No one had ever seen such a stone. Gold and silver idols beyond estimate were taken. One gold idol was ordered to be broken and 98300 mithgals of gold was found therein. In this way much property and many stones were captured… When Mahmud reached Ghazni, the booty was valued at 2000000 Dinars, 53000 slaves and 350 elephants.

$y_{6,4}$ → When Mahmud heard this he planned to go to that city and destroy the idols. From Hindustan he now set his force towards Somnath. When he approached the city and was seen by the Brahmanas and Sramanas, they all busied themselves in worshipping their idols. Before many days had passed a breach was effected, Mahmud's army entered the city and began to kill. Many Hindus were killed. Mahmud asked the Muazzan to go to the camp and announce the time of prayers. As he announced the call to prayers, all the idols were broken, burnt or otherwise destroyed. The stone idol of Manar was dug out from its foundation in the ground and broken into small pieces. Some of these, were taken to Ghazni on camels where they are still found under the steps of the Mosque. There was some treasure under the idols. All that treasure was taken. A large amount of property was thus got-silver idols, jewels, and treasure of various kinds.

The author Sidney Owen is a famous historian and he has been referred by the pupil. We considered his weightage as $\hat{w}_6 = 7$.

y_7 → Mohammad Habib[21]; $y_{7,1}$ → Northern India had ceased to attract Mahmud, for the spoils of its most wealthy temples were already in his treasury. But the rich and prosperous province of Gujrat was still untouched, and on October 18, 1025, he started from Ghaznin with his regular troops and thirty thousand volunteer-horsemen for the temple of Somnath, situated at the distance of a bow-shot from the mouth of the Saraswati, by the side of which the earthly body of Lord Krishna had breached its last.

$y_{7,2}$ → The temple was a spacious edifice and its roof was supported by fifty six ornamented columns. The idol was cut out of stone; it was five yards long, of which two yards were below, and three above ground. The Somnath expedition is the one by which Mahmud is most remembered. It was the finest achievement of his military genius. His marches into Hindustan hitherto had been through a fertile territory and he was never in danger of starvation.

$y_{7,3}$ → Mahmud entered the temple and possessed himself of its fabulous wealth. 'Not a hundredth part of the gold and precious stones he obtained from Somnath were to be found in the treasury of any king of Hindustan.'

Later historians have related how Mahmud refused the enormous ransom offered by the Brahmans, and preferred the title of 'Idol-breaker' (But-shikan) to that of 'idol-seller' (But-farosh).

And the author Mohammad Habibh is a well known historian, scholar. He has been referred by the researchers and pupil and we consider his weightage as $\hat{w}_5 = 8$.

The weights are normalized as follow, $\hat{w} = (\hat{w}_1\ \hat{w}_2\ \hat{w}_3\ \hat{w}_4\ \hat{w}_5\ \hat{w}_6\ \hat{w}_7)$; $\hat{w} = (9\ 9\ 7\ 8\ 9\ 7\ 8)$, $w = \frac{\hat{w}_i}{\sum_{i=1}^{k} \hat{w}_i} = (w_1\ w_2\ w_3\ w_4\ w_5\ w_6\ w_7)$. $w_1 = 0.158$; $w_2 = 0.158$; $w_3 = 0.123$; $w_4 = 0.140$; $w_5 = 0.158$; $w_6 = 0.123$; $w_7 = 0.140$, and the weightage of sub attributes are equal according to the attributes, $w_{11} = w_{12} = w_{13} = \frac{1}{3} = 0.333$; $w_{21} = w_{22} = w_{23} = \frac{1}{3} = 0.333$; $w_{31} = w_{32} = w_{33} = \frac{1}{3} = 0.333$; $w_{41} = w_{42} = w_{43} = \frac{1}{3} = 0.333$; $w_{51} = w_{52} = w_{53} = w_{54} = \frac{1}{4} = 0.25$; $w_{61} = w_{62} = w_{63} = w_{64} = \frac{1}{4} = 0.25$; $w_{71} = w_{72} = w_{73} = \frac{1}{3} = 0.333$

4.1.3 Alternatives

The most conflicting information regarding the purpose of 'Mahmud's raid on India' has been considered as alternatives a_r which has been interpreted by various aspects. Let the alternatives $a_r, r = 1, 2, \ldots, 6$ are defined as follow, $a_1 \rightarrow$ intention of the war was to destroy temple, idols and holy war; $a_2 \rightarrow$ to plunder and carry away gold and wealth; $a_3 \rightarrow$ to expand his kingdom; $a_4 \rightarrow$ to spread Islamism in India; $a_5 \rightarrow$ to exterminate the Hinduism; $a_6 \rightarrow$ just a myth

4.2 Aggregating assessment for Invasion of Mahmud of Ghazni on India

We have to construct the generalized decision matrix for the historical event on "Invasion of 'Mahmud of Ghazni' on India". In *Table 6.1*, the short forms are like EF=Extremely False; SF=Strongly False; F=False; MF=Moderately False; NI=Not Infer; MT=Moderately True; T=True; ST=Strongly True and ET=Extremely True.

Attributes		**Alternatives**					
Factors		a_1	a_2	a_3	a_4	a_5	a_6
y_1	y_{11}	[SF, .7]	[ST, .5];[T, .5]	[MF, .8]	[SF, .6];[NI, .1]	[EF, .8]	[EF, .9]
	y_{12}	[NI, .6]	[ET, .9]	[T, .5]	[F, .7]	[EF, .6];[SF, .1]	[SF, .8]
	y_{13}	[MT, .5]	[MF, .5]	[MT, .6]	[T, .7]	[MT, .5]	[MT, .6];[NI, .2]
y_2	y_{21}	[NI, .5];[MT, .1]	[NI, .5]	[MT, .6]	[F, .7]	[MT, .6]	[F,.8]
	y_{22}	[F, .7]	[MT, .5]	[ET, .8]	[EF, .5];[NI, .3]	[F, .7]	[NI, .5]
	y_{23}	[T, .5]	[MT, .4];[NI, .3]	[NI, .5];[MT, .2]	[T, .6];[MT, .2]	[T, .8]	[MT, .5];[NI, .2]

y_3	y_{31}	[T, .5]	[ET, .9]	[NI, .5]	[MT, .2];[NI, .5]	[MT, .5];[NI, .2]	[NI, .5];[F, .2]
	y_{32}	[MT, .4];[NI, .2]	[ET, .9]	[NI, .5]	[NI, .6]	[NI, .5]	[MT, .4]
	y_{33}	[MT, .5];[NI, .2]	[ET, .9]	[NI, .5];[MT, .4]	[F, .4];[NI, .2]	[MT, .3];[NI, .4]	[F, .5]
y_4	y_{41}	[SF, .8]	[NI, .6]	[ET, .9]	[SF, .7]	[SF, .7];[NI, .2]	[F, .6]
	y_{42}	[F, .5];[NI, .3]	[ET, .7]	[MT, .4];[NI, .2]	[SF, .4];[NI, .2]	[SF, .4];[NI, .3]	[F, .4]
	y_{43}	[T, .5]	[T, .5]	[ET, .7]	[NI, .5];[MT, .2]	[T, .3]	[F, .5]
y_5	y_{51}	[T, .5]	[ET, .7]	[NI, .5];[MT, .1]	[MT, .5]	[MT, .5]	[F, 3]
	y_{52}	[T, .4]	[MT, .4];[NI, .3]	[NI, .5]	[NI, .4];[F, .2]	[MT, .3];[NI, .3]	[SF, .4]
	y_{53}	[SF, .6]	[NI, .5]	[NI, .5]	[NI, .5]	[NI, .5]	[NI, .2];[MT, .3]
	y_{54}	[NI, .6]	[ET, .5]	[NI, .4]	[MF, .3];[NI, .5]	[F, .4];[NI, .3]	[F, .2];[NI, .3]
y_6	y_{61}	[MT, .2];[NI, .3]	[ET, .7]	[ET, .7]	[F, .5];[NI, .2]	[SF, .5]	[MF, .3];[NI, .3]
	y_{62}	[T, .5]	[ST, .6]	[NI, .7]	[T, .6]	[ST, .4];[T, .3]	[NI, .3];[F,.2]
	y_{63}	[T, .6]	[ET, .6]	[NI, .7]	[NI, .6]	[NI, .5];[MT, .1]	[NI, .3]
	y_{64}	[T, .7];[ST, .2]	[ET, .9]	[MT, .1];[NI, .3]	[T, .5];[NI, .4]	[T, .7]	[MT, .3]
y_7	y_{71}	[MT, .4];[NI, .3]	[ET, .6]	[NI, .4]	[F, .3]	[NI, .4]	[NI, .5]
	y_{72}	[NI, .5]	[T, .3];[NI, .5]	[NI, .5]	[NI, .3]	[NI, .4]	[MF, .2];[MT, .2]
	y_{73}	[T, .5]	[ST, .5]	[NI, .6]	[F, .3];[NI, .3]	[MT, .1];[NI, .3]	[F, .2]

Table 1: Generalized decision matrix for the historical event on "Invasion of 'Mahmud of Ghazni' on India"

Aggregating assessment for $y_1(a_1)$

Let $y_1 = y_{1,1} \oplus y_{1,2} \oplus y_{1,3}$, where $\oplus$ denotes the aggregation of two attributes. $\beta_{1,1} = 0$; $\beta_{2,1} = 0.7$; $\beta_{3,1} = 0$; $\beta_{4,1} = 0$; $\beta_{5,1} = 0$; $\beta_{6,1} = 0$; $\beta_{7,1} = 0$; $\beta_{8,1} = 0$; $\beta_{9,1} = 0$; $\beta_{1,2} = 0$; $\beta_{2,2} = 0$; $\beta_{3,2} = 0$; $\beta_{4,2} = 0$; $\beta_{5,2} = 0.6$;

$\beta_{6,2} = 0$; $\beta_{7,2} = 0$; $\beta_{8,2} = 0$; $\beta_{9,2} = 0$; $\beta_{1,3} = 0$; $\beta_{2,3} = 0$; $\beta_{3,3} = 0$; $\beta_{4,3} = 0$; $\beta_{5,3} = 0$; $\beta_{6,3} = 0.5$; $\beta_{7,3} = 0$; $\beta_{8,3} = 0$; $\beta_{9,3} = 0$. And the three factors are of equal importance, then $w_{11} = w_{12} = w_{13} = \frac{1}{3} = 0.333$.

And we can calculate the basic probability masses $m_{n,i}$ as follow: $m_{1,1} = 0$; $m_{2,1} = 0.2331$; $m_{3,1} = 0$; $m_{4,1} = 0$; $m_{5,1} = 0$; $m_{6,1} = 0$; $m_{7,1} = 0$; $m_{8,1} = 0$; $m_{9,1} = 0$; $\bar{m}_{H,1} = 0.667$; $\tilde{m}_{H,1} = 0.0999$; $m_{1,2} = 0$; $m_{2,2} = 0$; $m_{3,2} = 0$; $m_{4,2} = 0$; $m_{5,2} = 0.1998$; $m_{6,2} = 0$; $m_{7,2} = 0$; $m_{8,2} = 0$; $m_{9,2} = 0$; $\bar{m}_{H,2} = 0.667$; $\tilde{m}_{H,2} = 0.1332$; $m_{1,3} = 0$; $m_{2,3} = 0$; $m_{3,3} = 0$; $m_{4,3} = 0$; $m_{5,3} = 0$; $m_{6,3} = 0.1665$; $m_{7,3} = 0$; $m_{8,3} = 0$; $m_{9,3} = 0$; $\bar{m}_{H,3} = 0.667$;$\tilde{m}_{H,3} = 0.1665$. Now, we use the recursive equations $(5a) - (5e)$ to calculate the combined probability masses as follow: Let $m_{n,I(1)} = m_{n,1}$ for $n = 1, 2, \ldots, 9$. First we aggregate $y_{1,1}$ and $y_{1,2}$.

Since, $$K_{I(2)} = \left[1 - \sum_{t=1}^{9} \sum_{\substack{j=1 \\ j \neq t}}^{9} m_{t,I(1)} m_{j,2}\right]^{-1}.$$

$K_{I(2)} = 1.0489$ and $m_{H,1} = \bar{m}_{H,1} + \tilde{m}_{H,1} = 0.7669$; $m_{H,2} = \bar{m}_{H,2} + \tilde{m}_{H,2} = 0.8002$; $m_{H,3} = \bar{m}_{H,3} + \tilde{m}_{H,3} = 0.8335$

$m_{1,I(2)} = 0$; $m_{2,I(2)} = 0.1956$; $m_{3,I(2)} = 0$; $m_{4,I(2)} = 0$; $m_{5,I(2)} = 0.1607$; $m_{6,I(2)} = 0$; $m_{7,I(2)} = 0$; $m_{8,I(2)} = 0$; $m_{9,I(2)} = 0$; $\tilde{m}_{H,I(2)} = 0.1769$; $\bar{m}_{H,I(2)} = 0.4666$; $m_{H,I(2)} = 0.6435$. Now, we combine the above results for $y_{1,1}$ and $y_{1,2}$ with $y_{1,3}$ $K_{I(3)} = \left[1 - \sum_{t=1}^{9} \sum_{\substack{j=1 \\ j \neq t}}^{9} m_{t,I(2)} m_{j,3}\right]^{-1} = 1.063$.

$m_{1,I(3)} = 0$; $m_{2,I(3)} = 0.1733$; $m_{3,I(3)} = 0$; $m_{4,I(3)} = 0$; $m_{5,I(3)} = 0.1424$; $m_{6,I(3)} = 0.1139$; $m_{7,I(3)} = 0$; $m_{8,I(3)} = 0$; $m_{9,I(3)} = 0$; $\tilde{m}_{H,I(3)} = 0.2392$; $\bar{m}_{H,I(3)} = 0.3308$; $m_{H,I(3)} = 0.57$.

From the equations $(6a)$ and $(6b)$, the combined degrees of belief are calculated by $\beta_n = \frac{m_{n,I(3)}}{1 - \bar{m}_{H,I(3)}}$

$\beta_1 = \frac{m_{1,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0}{1-0.3308} = 0$; $\beta_2 = \frac{m_{2,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0.1733}{1-0.3308} = 0.259$; $\beta_3 = \frac{m_{3,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0}{1-0.3308} = 0$;

$\beta_4 = \frac{m_{4,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0}{1-0.3308} = 0$; $\beta_5 = \frac{m_{5,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0.1424}{1-0.3308} = 0.2128$; $\beta_6 = \frac{m_{6,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0.1139}{1-0.3308} = 0.1702$;

$\beta_7 = \frac{m_{7,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0}{1-0.3308} = 0$; $\beta_8 = \frac{m_{8,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0}{1-0.3308} = 0$; $\beta_9 = \frac{m_{9,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0}{1-0.3308} = 0$;

$\beta_H = \frac{\tilde{m}_{H,I(3)}}{1-\bar{m}_{H,I(3)}} = \frac{0.2392}{1-0.3308} = 0.3576$

The assessment for $y_1(a_1)$ is aggregating by $y_{1,1}$, $y_{1,2}$ and $y_{1,3}$ is given by the following distribution (7),

$S(y_1(a_1)) = S(y_{1,1} \oplus y_{1,2} \oplus y_{1,3}) =$
$\{(Strongly\ False, 0.259), (Not\ Infer, 0.2128), (Moderately\ True, 0.1702)\}$

We have produced the this algorithm for all alternatives as the same, and we get the assessment aggregation for all $y_2(a_1)$, $y_3(a_1)$, $y_4(a_1)$, $y_5(a_1)$, $y_6(a_1)$, and $y_7(a_1)$. Then finally we produced this algorithm for the final assessment of $y(a_1)$ is aggregating by y_1, y_2, y_3, y_4, y_5, y_5 and y_7 is given by the follow:

$S(y(a_1)) = S(y_1 \oplus y_2 \oplus y_3 \oplus y_4 \oplus y_5 \oplus y_6 \oplus y_7) =$
$\{(Strongly\ False, 0.1192), (False, 0.0716), (Not\ Infer, 0.2008),$
$(Moderately\ True, 0.1141), (True, 0.2389), (Strongly\ True, 0.0075)\}$

.

Suppose the $u(H)$, are as follow, $u(H_1) = 0$, $u(H_2) = 0.125$, $u(H_3) = 0.25$, $u(H_4) = 0.375$, $u(H_5) = 0.5$, $u(H_6) = 0.625$, $u(H_7) = 0.75$, $u(H_8) = 0.875$ and $u(H_9) = 1$. Then the degrees of belief for alternative a_1(intention of the war was to destroy temple, idols and holy war) are considered by the above assessments as follow: $\beta_1 = 0.0$; $\beta_2 = 0.1192$; $\beta_3 = 0.0716$; $\beta_4 = 0.0$; $\beta_5 = 0.2008$; $\beta_6 = 0.1141$; $\beta_7 = 0.2389$; $\beta_8 = 0.0075$; $\beta_9 = 0.0$ and $\beta_H = 0.3020$.

Since β_H is not zero, the assessment for $y(a_1)$ performance is not unique and is characterized by utility interval $[u_{min}(y(a_1)), u_{max}(y(a_1))]$ where $u_{min}(y(a_1)) = (\beta_1 + \beta_H)u(H_1) + \sum_{n=2}^{9} \beta_n \times u(H_n)$; $u_{min}(y(a_1)) = 0.3902$; $u_{max}(y(a_1)) = \sum_{n=1}^{8} \beta_n \times u(H_n) + (\beta_9 + \beta_H)u(H_9)$; $u_{max}(y(a_1)) = 0.6922$; $u_{avg}(y(a_1)) = \frac{(u_{min}(y(a_1)) + u_{max}(y(a_1)))}{2}$; $u_{avg}(y(a_1)) = 0.5412$.

Similarly we have produced this algorithm for all the alternatives till $y(a_6)$ and by the above calculation we ranked the alternatives as follow:

Alternatives	Maximum belief degree	Minimum belief degree	Average of belief degree	Rank for alternatives (Max. belief degree)
a_1	0.6922	0.3902	0.5412	3
a_2	0.9809	0.8049	0.8929	1
a_3	0.7782	0.5356	0.6569	2
a_4	0.6028	0.3445	0.4736	5
a_5	0.6879	0.3461	0.517	4
a_6	0.5920	0.2491	0.4206	6

Table 2 Ranked for Alternatives $y(a)$

Therefore the preferences for the alternatives are, the Mahmud's war to plunder and carry away gold and wealth > to expand his kingdom > intention of the war was to destroy temple, idols and holy war > to exterminate the Hinduism > to spread Islamism in India > just a myth.

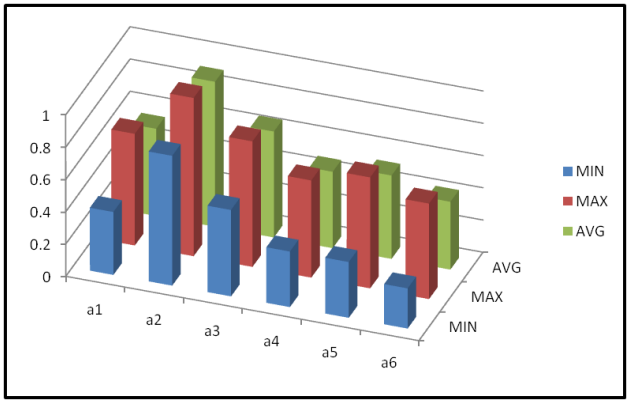

Figure 2: Utility Intervals for Alternatives

V. CONCLUSION

The reports on the historical event "INVASION OF MAHMUD OF GHAZNI ON INDIA IN 1001 – 1027 AD" has been studied in this dissertation which posses numerous rumors. We considered the 'rumors' as alternatives. Also we analyze the reports on the same event by different authors from various backgrounds like religion, country, and so on.

Using Evidential Reasoning approach, we rank the alternativeness in accordance with the one which is closer to reality. As per the result obtained, we conclude that the maximum reason for invasion was **to plunder and carry away gold and wealth** (a_2). The secondly preferred attribute was **to expand his kingdom** (a_3). The attributes, **intention of the war was to destroy temple, idols and holy war** (a_1) and **to exterminate the Hinduism** (a_5) were supportive reasons for the war. The attributes ***to spread Islamism in India*** (a_4) and ***just a myth*** (a_6) are ranked the least.

Hence through Evidential Reasoning approach we conclude that the author's A. L. Basham, W. W. Hunter, Romila Thapar were reporting the invasion, primarily as an act for Mahmud's insatiable desire for gold and wealth rather than to spread Islam.

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