NEW ITERATIVE METHOD FOR SOLVING HIGHER ORDER KDV EQUATIONS

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ABSTRACT

Generalized Korteweg-de Vries equation of fifth order (gfKdV) and seventh order (gsKdV) has various applications in Sciences and Engineering. In this paper, a New Iterative Method (NIM) is being proposed to obtain the solution of several forms of the gfKdV and gsKdV equations. We have shown that the NIM solution is more accurate as comparedto the techniques such as, homotopy perturbation method and adomian decomposition method. Further, results also demonstrate that NIM solution is more reliable, easy to compute and computationally fast as compared to other methods.

Keywords: Korteweg-deVries eq.s, New Iterative Method, Adomian Decomposition Method, Homotopy Perturbation Method.

1. INTRODUCTION

The generalized KdV equation of fifth-order (gfKdV) is defined as:

$$u_t + a\,u^2 u_x + b u_x u_{2x} + c u u_{3x} + d u_{5x} = 0, \qquad (1)$$

where a, b, c and d are arbitrary non zero real parameters and the subscripts denote the derivatives of the corresponding variable. The fifth-order KdVequationplay vital role in various domains such as describing motion of the long waves in shallow water under gravity,conformal field theory, two-dimensional quantum gravitationcanonical field theory, nonlinear optics etc.

Generalized KdV equation of seventh-order (gsKdV) is written as:

$$u_t + a u^3 u_x + b u_x^3 + c u u_x u_{2x} + d u^2 u_{3x} + e u_{2x} u_{3x} + f u_x u_{4x} + g\, u u_{5x} + u_{7x} = 0, \qquad (2)$$

where a, b, c, d, e, f and $g$are arbitrary non zero parameters. This equation plays an important role in mathematical physics, engineering and applied sciences for investigating travelling solitary wave solutions.

Various numerical techniques have been proposed in the past to solve these equations. Some of the popular techniques for fifth and seventh order KdV equations are Adomian decomposition method [1], modified Adomian decomposition method [2], variational iteration method [3], modified variational iteration method, homotopy perturbation method, modified homotopy perturbation method and homotopy analysis method [4], Expfunction method [5], homogeneous balance method [6], extended tanh method [7] etc. In addition to these techniques the seventh order KdVequations are also being solved using the Hirota direct method and the tanh-coth method [8], homotopy perturbation method [9, 10], variational iteration method [11,12], homotopy analysis method [13], Adomian decomposition method [14], Cole-Hopf transformation [15] and reconstruction of variational iteration method [16].

In the present paper, we employ New Iterative Method (NIM), developed by DaftardarGejji and Jafari [17], to solve generalized Korteweg-de Vries equations of fifth and seventh orders [18, 19]. NIM has been used by many researchers to solve linear and nonlinear equations of integer and fractional orders [20, 21, 22, 23]. Advantage of NIM is that it gives highly accurate solution with comparatively much lesser number of iterations. Further, it does not involve additional overhead in computing terms such as*adomian polynomials* in ADM[24,14,25] and*construction of homotopy function* in HPM [10, 26].

The organization of this paper is as follows: In section 2, we give the basic introduction of NIM. Solutions of the generalized KdV equations using NIMderived for several forms of gfKdV and gsKdVare discussed in section 3. We call these solutions as NIM solutions.NIM solutionsare accurate and the NIM computation technique is faster as compared to the other commonly used techniques such as HPM[10] and ADM[14]. The comparisonsbetween the numerical results of the proposed NIM solutions with that ofHPM and ADMare discussed in section 4. Finally,we conclude by summarizing the advantages of using NIM in solving higher order KdV equations in section 4.

II NEW ITERATIVE METHOD (NIM)

To illustrate the idea of the NIM, we consider the following general functional equation:

$$u = f + N(u), \tag{3}$$

where N is a nonlinear operator from a Banach space B → B and f is a known function. We are looking for a solution u of (3) having the series form

$$u = \sum_{i=0}^{\infty} u_i. \tag{4}$$

The nonlinear operator N can be decomposed as:

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty}\left[N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right] \tag{5}$$

Now using the above eq.s (4) and (5) in (3):

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\left[N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right] \tag{6}$$

We define the recurrence relation in the following way:

$$\begin{aligned} u_1 &= N(u_0) \\ u_2 &= N(u_0 + u_1) - N(u_0) \\ u_3 &= N(u_0 + u_1 + u_2) - N(u_0 + u_1) \\ u_{n+1} &= N(u_0 + u_1 + \cdots u_n) - N(u_0 + u_1 + \cdots u_{n-1}); \qquad n = 1, 2, 3, \ldots \end{aligned} \tag{7}$$

Then,

$$u_1 + u_2 + \cdots + u_{n+1} = N(u_0 + u_1 + \cdots + u_n), \qquad n = 1, 2, 3, \ldots \tag{8}$$

and

$$\sum_{i=0}^{\infty} u_i = f + N\left(\sum_{j=0}^{\infty} u_j\right). \tag{9}$$

The m-term approximate solution of (3) is given by $u \approx u_0 + u_1 + u_2 + \cdots + u_{m-1}$. For understanding the convergence of this method we refer reader to [27].

Solution of higher order KdV equations using NIM

In this section, we discuss solution of higher order KdV equations using NIM. For this we consider two scenarios each, for fifth order and seventh order Korteweg-deVries (KdV) equations.

2.1 Lax and Sawada-Kotera fifth order equations

Equation (1), as in section 1, is known as Lax fifth order KdV for a = 30, b = 30, c = 10 and d = 1 (Example 1) and Sawada-Kotera fifth order KdVs for a = 45, b = 15, c = 15 and d = 1 (Example 2). We next derive their solutions using NIM. In the following discussion we denote $J_t[U] = \int_0^t (U)dt$.

Example1: The Lax fifth-order KdV equation [10, 6, 29]:

$$u_t + 30\, u^2 u_x + 30 u_x u_{2x} + 10 u\, u_{3x} + u_{5x} = 0, \qquad (10)$$

with the initial condition:

$$u(x,0) = 2k^2(2 - 3\tanh[k(x - \mathrm{x0})]^2), \qquad (11)$$

has an exact solution:

$$u(x,t) = 2k^2(2 - 3\tanh[k(x - 56k^4 t - \mathrm{x}_0)]^2). \qquad (12)$$

Considering the equivalent integral equation for (10-11):

$$u(x,t) = u(x,0) + J_t[-(30\, u^2 u_x + 30 u_x u_{2x} + 10 u\, u_{3x} + u_{5x})]. \qquad (13)$$

On comparing it with equation (3),we have $N(u) = J_t[-(30\, u^2 u_x + 30 u_x u_{2x} + 10 u\, u_{3x} + u_{5x})]$

The NIMrecurrence relation for Lax fifth-order KdV equation (10), using its equivalent form (13) and the general solution (7) is:

$$u_1 = N(u_0)$$

$$u_1 = 6k^7 t\,\mathrm{Sech}[k(x - \mathrm{x0})]^7(-586\mathrm{Sinh}[k(x - \mathrm{x0})] + 141\mathrm{Sinh}[3k(x - \mathrm{x0})] + 7\mathrm{Sinh}[5k(x - \mathrm{x0})]) \qquad (14)$$

And, its2-term NIM solution (i.e. $u \approx u_0 + u_1$)usinginitial condition(11) is:

$$u \approx 2k^2(2 - 3\tanh[k(x - \mathrm{x0})]^2) + 6k^7 t\,\mathrm{Sech}[k(x - \mathrm{x0})]^7(-586\mathrm{Sinh}[k(x - \mathrm{x0})]) +$$

$$6k^7 t\,\mathrm{Sech}[k(x - \mathrm{x0})]^7(141\mathrm{Sinh}[3k(x - \mathrm{x0})] + 7\mathrm{Sinh}[5k(x - \mathrm{x0})]). \qquad (15)$$

Example 2: The Sawada-Kotera fifth order KdV eq. [10, 6, 28]

$$u_t + 45\, u^2 u_x + 15 u_x u_{2x} + 15 u\, u_{3x} + u_{5x} = 0, \qquad (16)$$

with the initial condition:

$$u(x,0) = 2k^2\mathrm{Sech}[k(x - \mathrm{x0})]^2, \qquad (17)$$

has an exact solution:

$$u(x,t) = 2k^2\mathrm{Sech}[k(-16k^4 t + x - \mathrm{x}_0)]^2.$$

The initial value problem (16-17) is equivalent to

$$u(x,t) = u(x,0) + J_t[-(45\, u^2 u_x + 15 u_x u_{2x} + 15 u\, u_{3x} + u_{5x})] \qquad (18)$$

On comparing it with equation (3), we have $N(u) = J_t[-(45\, u^2 u_x + 15 u_x u_{2x} + 15 u\, u_{3x} + u_{5x})]$.

In view of the recurrence relation (7), we get

$$u_1 = N(u_0) = 64k^7 t\text{Sech}[k(x-\text{x0})]^2\text{Tanh}[k(x-\text{x0})] \quad (19)$$

$$u_2 = N(u_0+u_1) - N(u_0)$$

$$u_2 = 8k^{12}t^2\text{Sech}[k(x-\text{x0})]^{10}(-25+460800k^{10}t^2-2(17+276480k^{10}t^2)\text{Cosh}[2k(x-\text{x0})]+(-8+92160k^{10}t^2)\text{Cosh}[4k(x-\text{x0})]+2\text{Cosh}[6k(x-\text{x0})]+\text{Cosh}[8k(x-\text{x0})]+29440k^5t\text{Sinh}[2k(x-\text{x0})]-14080k^5t\text{Sinh}[4k(x-\text{x0})]+1280k^5t\text{Sinh}[6k(x-\text{x0})])$$

. (20)

Therefore, 3-term NIM solution of (16) (i.e., $u \approx u_0+u_1+u_2$) with initial condition (17) is:

$$u \approx 2k^2\text{Sech}[k(x-\text{x0})]^2 + 8k^{12}t^2\text{Sech}[k(x-\text{x0})]^{10}\times$$

$$(-25+460800k^{10}t^2-2(17+276480k^{10}t^2)\text{Cosh}[2k(x-\text{x0})]+(-8+92160k^{10}t^2)\text{Cosh}[4k(x-\text{x0})]+2\text{Cosh}[6k(x-\text{x0})]+\text{Cosh}[8k(x-\text{x0})]+29440k^5t\text{Sinh}[2k(x-\text{x0})]-14080k^5t\text{Sinh}[4k(x-\text{x0})]+1280k^5t\text{Sinh}[6k(x-\text{x0})])+64k^7t\text{Sech}[k(x-\text{x0})]^2\text{Tanh}[k(x-\text{x0})].$$

(21)

2.2 Lax's and Sawada-Kotera seventh order KdV equations

In this section, we solve generalized seventh order Korteweg-de Vries (KdV) equations (2) using NIM for a=140,b = 70,c = 280,d = 70,e = 70,f = 42 and g = 14 (Lax seventh order KdV equation) and for a = 252,b = 63,c = 378,d = 126,e = 63,f = 42 and g = 21 (Sawada-Kotera seventh order KdV equation).

Example3: The Lax seventh order KdV eq. [14, 29]

$$u_t + 140u^3u_x + 70u_x^3 + 280uu_xu_{2x} + 70u^2u_{3x} + 70u_{2x}u_{3x} + 42u_xu_{4x} + 14uu_{5x} + u_{7x} = 0 \quad (22)$$

with the initial condition:

$$u(x,0) = 2k^2\text{Sech}[kx]^2, \quad (23)$$

hasan exact solution

$$u(x,0) = 2k^2\text{Sech}[k(x-64k^6t)]^2. \quad (24)$$

The eq. (22) along with the initial condition (23) can be written equivalently as

$$u(x,t) = u(x,0) + J_t[-(140u^3u_x + 70u_x^3 + 280uu_xu_{2x} + 70u^2u_{3x} + 70u_{2x}u_{3x} + 42u_xu_{4x} + 14uu_{5x} + u_{7x})] \quad (25)$$

where

$$N(u) = J_t[-(140u^3u_x + 70u_x^3 + 280uu_xu_{2x} + 70u^2u_{3x} + 70u_{2x}u_{3x} + 42u_xu_{4x} + 14uu_{5x} + u_{7x})] \quad (26)$$

Now using the recurrence relation (7) in (26):

$$u_1 = N(u_0) = 256k^9t\text{Sech}[kx]^2\text{Tanh}[kx] \quad (27)$$

$$u_2 = N(u_0+u_1) - N(u_0)$$

$$u_2 = \frac{16}{3}k^{16}t^2\text{Sech}[kx]^{13}(-882(1+1638400k^{14}t^2)\text{Cosh}[kx]+6(-87+318832640k^{14}t^2)\text{Cosh}[3kx]-153\text{Cosh}[5kx]-564264960k^{14}t^2\text{Cosh}[5kx]+3\text{Cosh}[7kx]+41287680k^{14}t^2\text{Cosh}[7kx]+15\text{Cosh}[9kx]+3\text{Cosh}[11kx]-30994432k^7t\text{Sinh}[kx]+45097156608k^{21}t^3\text{Sinh}[kx]-15009792k^7t\text{Sinh}[3kx]-19730006016k^{21}t^3\text{Sinh}[3kx]+13834240k^7t\text{Sinh}[5kx]+2818572288k^{21}t^3\text{Sinh}[5kx]-2085888k^7t\text{Sinh}[7kx]+64512k^7t\text{Sinh}[9kx]).$$

(28)

Therefore, the 3-term approximate solution obtained by NIM of (22) with initial condition (23) is given by

$$u(x,t) \approx 2k^2\text{Sech}[kx]^2 + \frac{16}{3}k^{16}t^2\text{Sech}[kx]^{13}(-882(1+1638400\,k^{14}t^2)\text{Cosh}[kx] + 6(-87 + 318832640k^{14}t^2)\text{Cosh}[3kx] - 153\text{Cosh}[5kx] - 564264960\,k^{14}t^2\text{Cosh}[5kx] + 3\text{Cosh}[7kx] + 41287680k^{14}t^2\text{Cosh}[7kx] + 15\text{Cosh}[9kx] + 3\text{Cosh}[11kx] - 30994432k^7t\text{Sinh}[kx] + 45097156608k^{21}t^3\text{Sinh}[kx] - 15009792\,k^7t\text{Sinh}[3kx] - 19730006016\,k^{21}t^3\text{Sinh}[3kx] + 13834240k^7t\text{Sinh}[5kx] + 2818572288\,k^{21}t^3\text{Sinh}[5kx] - 2085888k^7t\text{Sinh}[7kx] + 64512k^7t\text{Sinh}[9kx]) + 256k^9t\text{Sech}[kx]^2\text{Tanh}[kx] \quad (29)$$

Example4: The Sawada-Kotera seventh order KdV eq. [14, 29]

$$u_t + 252\,u_x + 63u_x^3 + 378u\,u_xu_{2x} + 126\,u^2u_{3x} + 63u_{2x}u_{3x} + 42u_xu_{4x} + 21\,uu_{5x} + u_{7x} = 0 \quad (30)$$

with the initial condition:

$$u(x,0) = \frac{4}{3}k^2(2 - 3\text{Tanh}[kx]^2), \quad (31)$$

has an exact solution:

$$u(x,t) = \frac{4}{3}k^2(2 - 3\text{Tanh}[k(x - \frac{256k^6t}{3})]^2). \quad (32)$$

The eq. (30) along with the initial condition (31) can be written as

$$u(x,t) \approx u(x,0) + J_t[-(252u_x + 63u_x^3 + 378u\,u_xu_{2x} + 126\,u^2u_{3x} + 63u_{2x}u_{3x} + 42u_xu_{4x} + 21\,uu_{5x} + u_{7x})] \quad (33)$$

where,

$$N(u) = J_t[-(252u_x + 63u_x^3 + 378u\,u_xu_{2x} + 126u^2u_{3x} + 63u_{2x}u_{3x} + 42u_xu_{4x} + 21\,uu_{5x} + u_{7x})] \quad (34)$$

Using the recurrence relation (7), we get:

$$u_1 = N(u_0) = -\frac{2048}{3}k^9\text{Sech}[kx]^2\text{Tanh}[kx] \quad (35)$$

$$u_2 = N(u_0 + u_1) - N(u_0)$$

$$u_2 = \frac{512}{45}k^{16}t^2\text{Sech}[kx]^{13}(1470(-1 + 8126464\,k^{14}t^2)\text{Cosh}[kx] - 30(29 + 519307264k^{14}t^2)\text{Cosh}[3kx] - 255\text{Cosh}[5kx] + 4348968960k^{14}t^2\text{Cosh}[5kx] + 5\text{Cosh}[7kx] - 275251200k^{14}t^2\text{Cosh}[7kx] + 25\text{Cosh}[9kx] + 5\text{Cosh}[11kx] + 1924145348608\,k^{21}t^3\text{Sinh}[kx] - 841813590016\,k^{21}t^3\text{Sinh}[3kx] + 120259084288\,k^{21}t^3\text{Sinh}[5kx]) \quad (36)$$

Therefore, the 3-term approximate NIM solution of (30-31) (i.e., $u \approx u_0 + u_1 + u_2$) is given by:

$$u \approx \frac{512}{45}k^{16}t^2\text{Sech}[kx]^{13}(1470(-1 + 8126464\,k^{14}t^2)\text{Cosh}[kx] - 30(29 + 519307264\,k^{14}t^2)\text{Cosh}[3kx] - 255\text{Cosh}[5kx] + 4348968960k^{14}t^2\text{Cosh}[5kx] + 5\text{Cosh}[7kx] - 275251200k^{14}t^2\text{Cosh}[7kx] + 25\text{Cosh}[9kx] + 5\text{Cosh}[11kx] + 1924145348608\,k^{21}t^3\text{Sinh}[kx] - 841813590016\,k^{21}t^3\text{Sinh}[3kx] + 120259084288\,k^{21}t^3\text{Sinh}[5kx]) - \frac{2048}{3}k^9t\text{Sech}[kx]^2\text{Tanh}[kx] + \frac{4}{3}k^2(2 - 3\text{Tanh}[kx]^2) \quad (37)$$

III RESULT ANALYSIS

We discuss the numerical results of the NIM solutions for Lax equations and Sawada- Kotera equations of fifth and seventh orders as derived in section 3. Tables1 and 2 give the numerical comparison between HPM and NIM solutionsfor the fifth order equation,and Tables 3 and 4 are the comparison between ADM and NIM solutions for the seventh order. Graphical representations of NIM solutions and exact solutions for Lax equations of fifth and seventh order are shown in Figures 1 and 3 respectively. Similarly graphical comparison between NIM solutions and exact solutions of Sawada-Kotera fifth and seventh order equations are shown in Figures 2and 4 respectively.

x	t	Exact Solution	10-term HPM Abs Error	2-term NIM Abs Error
0.1	0.1	3.99999×10^{-4}	9.60×10^{-12}	1.44004×10^{-15}
	0.2	3.99999×10^{-4}	1.92×10^{-11}	2.88002×10^{-15}
	0.3	3.99999×10^{-4}	2.88×10^{-11}	4.32000×10^{-15}
	0.4	3.99999×10^{-4}	3.84×10^{-11}	1.44004×10^{-15}
	0.5	3.99999×10^{-4}	4.78×10^{-11}	1.44004×10^{-15}
0.2	0.1	3.99998×10^{-4}	9.60×10^{-11}	2.87991×10^{-15}
	0.2	3.99998×10^{-4}	1.92×10^{-11}	5.75982×10^{-15}
	0.3	3.99998×10^{-4}	2.88×10^{-11}	8.63979×10^{-15}
	0.4	3.99998×10^{-4}	3.84×10^{-11}	1.15198×10^{-14}
	0.5	3.99998×10^{-4}	4.80×10^{-11}	1.15198×10^{-14}
0.3	0.1	3.99995×10^{-4}	9.50×10^{-11}	4.31979×10^{-15}
	0.2	3.99995×10^{-4}	1.91×10^{-11}	8.63963×10^{-15}
	0.3	3.99995×10^{-4}	2.88×10^{-11}	1.29594×10^{-14}
	0.4	3.99995×10^{-4}	3.84×10^{-11}	1.72792×10^{-14}
	0.5	3.99995×10^{-4}	4.80×10^{-11}	1.72792×10^{-14}
0.4	0.1	3.99999×10^{-4}	9.50×10^{-11}	5.75944×10^{-15}
	0.2	3.99999×10^{-4}	1.91×10^{-11}	1.15190×10^{-14}
	0.3	3.99999×10^{-4}	2.87×10^{-11}	1.72786×10^{-14}
	0.4	3.99999×10^{-4}	3.87×10^{-11}	2.30380×10^{-14}
	0.5	3.99999×10^{-4}	4.79×10^{-11}	2.30380×10^{-14}
0.5	0.1	3.99985×10^{-4}	9.60×10^{-11}	7.19905×10^{-15}
	0.2	3.99985×10^{-4}	1.92×10^{-11}	1.43981×10^{-14}
	0.3	3.99985×10^{-4}	2.88×10^{-11}	2.15971×10^{-14}
	0.4	3.99985×10^{-4}	3.84×10^{-11}	2.87961×10^{-14}
	0.5	3.99985×10^{-4}	4.80×10^{-11}	2.87961×10^{-14}

Table 1. Comparison between 2-term NIM errorand10-term HPM errorfor eq.(10)($x_0 = 0.0, k = 0.01$)

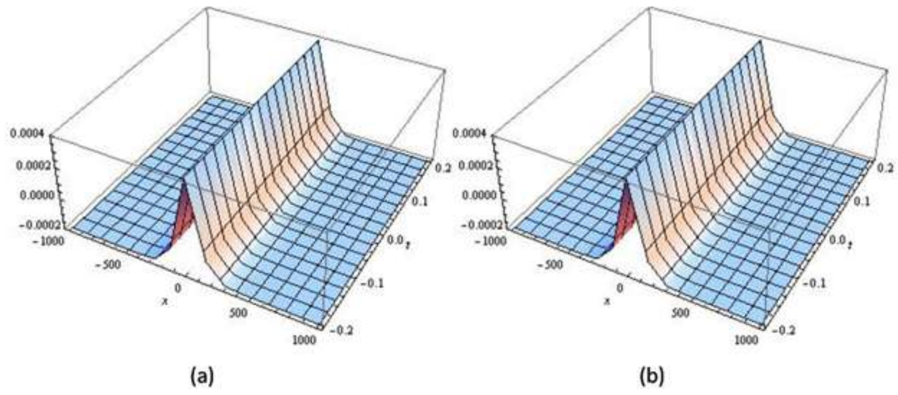


Figure1. For k = 0.01, $x_0 = 0$: (a) exact solution (b) 2-term NIM solution of (10).

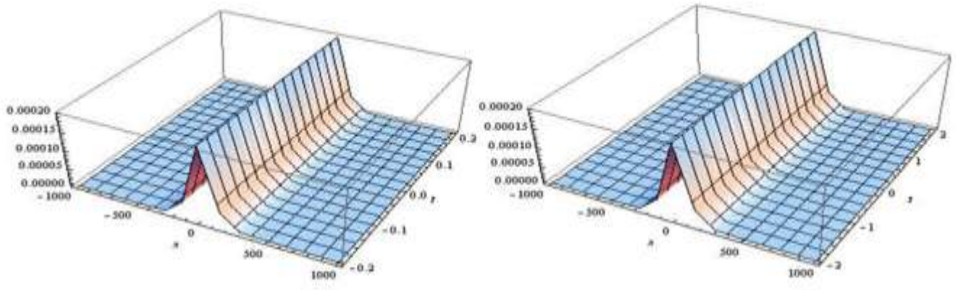
x	t	Exact Solution	10-term HPM Abs Error	2-term NIM Abs Error
0.1	0.1	1.999998×10^{-4}	4.800×10^{-16}	2.71051×10^{-20}
	0.2	1.999998×10^{-4}	9.600×10^{-16}	0
	0.3	1.999998×10^{-4}	1.440×10^{-15}	8.13152×10^{-20}
	0.4	1.999998×10^{-4}	1.920×10^{-15}	2.71051×10^{-20}
	0.5	1.999998×10^{-4}	2.400×10^{-15}	2.71051×10^{-20}
0.2	0.1	1.999992×10^{-4}	9.600×10^{-16}	0
	0.2	1.999992×10^{-4}	1.920×10^{-15}	2.71051×10^{-20}
	0.3	1.999992×10^{-4}	2.880×10^{-15}	2.71051×10^{-20}
	0.4	1.999992×10^{-4}	3.840×10^{-15}	2.71051×10^{-20}
	0.5	1.999992×10^{-4}	4.800×10^{-15}	2.71051×10^{-20}
0.3	0.1	1.999982×10^{-4}	1.440×10^{-15}	2.71051×10^{-20}
	0.2	1.999982×10^{-4}	2.880×10^{-15}	5.42101×10^{-20}
	0.3	1.999982×10^{-4}	4.320×10^{-15}	0
	0.4	1.999982×10^{-4}	5.760×10^{-15}	0
	0.5	1.999982×10^{-4}	7.200×10^{-15}	5.42101×10^{-20}
0.4	0.1	1.999968×10^{-4}	1.920×10^{-15}	2.71051×10^{-20}
	0.2	1.999968×10^{-4}	3.840×10^{-15}	8.13152×10^{-20}
	0.3	1.999968×10^{-4}	5.760×10^{-15}	2.71051×10^{-20}
	0.4	1.999968×10^{-4}	7.680×10^{-15}	2.71051×10^{-20}
	0.5	1.999968×10^{-4}	9.599×10^{-15}	5.42101×10^{-20}
0.5	0.1	1.99950×10^{-4}	2.400×10^{-14}	8.13152×10^{-20}
	0.2	1.99950×10^{-4}	4.800×10^{-14}	5.42101×10^{-20}
	0.3	1.99950×10^{-4}	7.200×10^{-14}	5.42101×10^{-20}
	0.4	1.99950×10^{-4}	9.600×10^{-14}	5.42101×10^{-20}
	0.5	1.99950×10^{-4}	1.200×10^{-14}	5.42101×10^{-20}

Table 2: Comparison between 3-term NIM errorand 10-term HPM errorfor eq.(16) (x_0 = 0.0,k = 0.01)

x	t	Exact Solution	5-term ADM Abs. Error	3-term NIM Abs Error
0.1	0.1	1.99980×10^{-2}	1.31265×10^{-8}	3.46945×10^{-18}
	0.2	1.99980×10^{-2}	2.46564×10^{-8}	0
	0.3	1.99980×10^{-2}	3.60575×10^{-8}	0
	0.4	1.99980×10^{-2}	4.72723×10^{-8}	0
	0.5	1.99980×10^{-2}	5.82447×10^{-8}	3.46945×10^{-18}
0.2	0.1	1.99920×10^{-2}	2.69869×10^{-8}	0
	0.2	1.99920×10^{-2}	4.99471×10^{-8}	3.46945×10^{-18}
	0.3	1.99920×10^{-2}	7.27082×10^{-8}	6.93889×10^{-18}
	0.4	1.99920×10^{-2}	9.51513×10^{-8}	3.46945×10^{-18}
	0.5	1.99920×10^{-2}	1.17157×10^{-7}	3.46945×10^{-18}
0.3	0.1	1.99820×10^{-2}	3.50406×10^{-8}	0
	0.2	1.99820×10^{-2}	6.95085×10^{-8}	3.46945×10^{-18}
	0.3	1.99820×10^{-2}	1.03998×10^{-8}	3.46945×10^{-18}
	0.4	1.99820×10^{-2}	1.38303×10^{-8}	3.46945×10^{-18}
	0.5	1.99820×10^{-2}	1.72205×10^{-8}	6.93889×10^{-18}
0.4	0.1	1.99680×10^{-2}	2.65114×10^{-8}	0
	0.2	1.99680×10^{-2}	7.29819×10^{-8}	3.46945×10^{-18}
	0.3	1.99680×10^{-2}	1.20366×10^{-7}	0
	0.4	1.99680×10^{-2}	1.68309×10^{-7}	3.46945×10^{-18}
	0.5	1.99680×10^{-2}	2.16406×10^{-7}	1.04083×10^{-17}
0.5	0.1	1.99550×10^{-2}	1.36124×10^{-8}	0
	0.2	1.99550×10^{-2}	4.60129×10^{-8}	3.46945×10^{-18}
	0.3	1.99550×10^{-2}	1.08644×10^{-7}	6.93889×10^{-18}
	0.4	1.99550×10^{-2}	1.73664×10^{-7}	6.93889×10^{-18}
	0.5	1.99550×10^{-2}	2.40318×10^{-7}	1.04083×10^{-18}
x	**t**	**Exact Solution**	**5-term ADM Abs. Error**	**3-term NIM Abs Error**
0.1	0.1	1.99980×10^{-2}	1.31265×10^{-8}	3.46945×10^{-18}
	0.2	1.99980×10^{-2}	2.46564×10^{-8}	0
	0.3	1.99980×10^{-2}	3.60575×10^{-8}	0
	0.4	1.99980×10^{-2}	4.72723×10^{-8}	0
	0.5	1.99980×10^{-2}	5.82447×10^{-8}	3.46945×10^{-18}
0.2	0.1	1.99920×10^{-2}	2.69869×10^{-8}	0
	0.2	1.99920×10^{-2}	4.99471×10^{-8}	3.46945×10^{-18}
	0.3	1.99920×10^{-2}	7.27082×10^{-8}	6.93889×10^{-18}
	0.4	1.99920×10^{-2}	9.51513×10^{-8}	3.46945×10^{-18}
	0.5	1.99920×10^{-2}	1.17157×10^{-7}	3.46945×10^{-18}
0.3	0.1	1.99820×10^{-2}	3.50406×10^{-8}	0

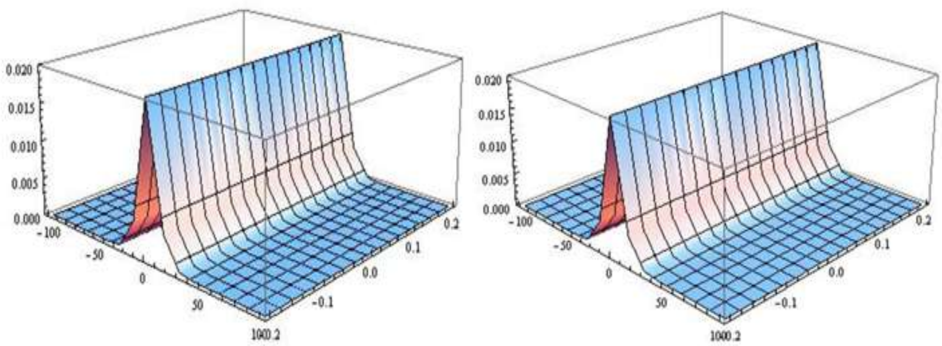
	0.2	1.99820×10^{-2}	6.95085×10^{-8}	3.46945×10^{-18}
	0.3	1.99820×10^{-2}	1.03998×10^{-8}	3.46945×10^{-18}
	0.4	1.99820×10^{-2}	1.38303×10^{-8}	3.46945×10^{-18}
	0.5	1.99820×10^{-2}	1.72205×10^{-8}	6.93889×10^{-18}
0.4	0.1	1.99680×10^{-2}	2.65114×10^{-8}	0
	0.2	1.99680×10^{-2}	7.29819×10^{-8}	3.46945×10^{-18}
	0.3	1.99680×10^{-2}	1.20366×10^{-7}	0
	0.4	1.99680×10^{-2}	1.68309×10^{-7}	3.46945×10^{-18}
	0.5	1.99680×10^{-2}	2.16406×10^{-7}	1.04083×10^{-17}
0.5	0.1	1.99550×10^{-2}	1.36124×10^{-8}	0
	0.2	1.99550×10^{-2}	4.60129×10^{-8}	3.46945×10^{-18}
	0.3	1.99550×10^{-2}	1.08644×10^{-7}	6.93889×10^{-18}
	0.4	1.99550×10^{-2}	1.73664×10^{-7}	6.93889×10^{-18}
	0.5	1.99550×10^{-2}	2.40318×10^{-7}	1.04083×10^{-18}

Table 3. Comparison between 3-term NIM errorand5-term ADM errorfor eq.(22) (x_0 = 0.0,k = 0.1)



(a) (b)

Figure2. For k = 0.01, x_0 = 0: (a) exact solution; (b) 3-term NIM solution of (16).



(a) (b)

Figure 3. For k = 0.1: (a) exact solution (b) 3-term NIM solution of (22).

x	t	Exact Solution	5-term ADM Abs. Error	2-term NIM Abs Error
0.1	0.1	2.66627×10^{-2}	6.61289×10^{-9}	1.36515×10^{-9}
	0.2	2.66627×10^{-2}	1.38091×10^{-8}	2.73030×10^{-9}
	0.3	2.66627×10^{-2}	2.10236×10^{-8}	4.09545×10^{-9}
	0.4	2.66627×10^{-2}	2.82628×10^{-8}	5.46061×10^{-9}
	0.5	2.66627×10^{-2}	3.55331×10^{-8}	6.82576×10^{-9}
0.2	0.1	2.66507×10^{-2}	1.30572×10^{-8}	2.72921×10^{-9}
	0.2	2.66507×10^{-2}	2.74896×10^{-8}	5.45842×10^{-9}
	0.3	2.66507×10^{-2}	4.19330×10^{-8}	8.18763×10^{-9}
	0.4	2.66507×10^{-2}	5.64022×10^{-8}	1.09168×10^{-8}
	0.5	2.66507×10^{-2}	7.09127×10^{-8}	1.36461×10^{-8}
0.3	0.1	2.66307×10^{-2}	2.20865×10^{-8}	4.09109×10^{-9}
	0.2	2.66307×10^{-2}	4.37197×10^{-8}	8.18218×10^{-9}
	0.3	2.66307×10^{-2}	6.52333×10^{-8}	1.22733×10^{-8}
	0.4	2.66307×10^{-2}	8.66610×10^{-8}	1.63644×10^{-8}
	0.5	2.66307×10^{-2}	1.08043×10^{-7}	2.04554×10^{-8}
0.4	0.1	2.66027×10^{-2}	3.82381×10^{-8}	5.44979×10^{-9}
	0.2	2.66027×10^{-2}	6.68601×10^{-8}	1.08994×10^{-8}
	0.3	2.66027×10^{-2}	9.49483×10^{-8}	1.63491×10^{-8}
	0.4	2.66027×10^{-2}	1.22581×10^{-7}	2.17988×10^{-8}
	0.5	2.66027×10^{-2}	1.49860×10^{-7}	2.72485×10^{-8}
0.5	0.1	2.65668×10^{-2}	6.78327×10^{-8}	6.80396×10^{-9}
	0.2	2.65668×10^{-2}	1.02954×10^{-7}	1.36079×10^{-8}
	0.3	2.65668×10^{-2}	1.36620×10^{-7}	2.04119×10^{-8}
	0.4	2.65668×10^{-2}	1.69003×10^{-7}	2.72158×10^{-8}
	0.5	2.65668×10^{-2}	2.00334×10^{-7}	3.40198×10^{-8}

Table 4. Comparison between 3-term NIM errorand5-term ADM errorfor eq.(30) ($x_0 = 0.0, k = 0.1$)

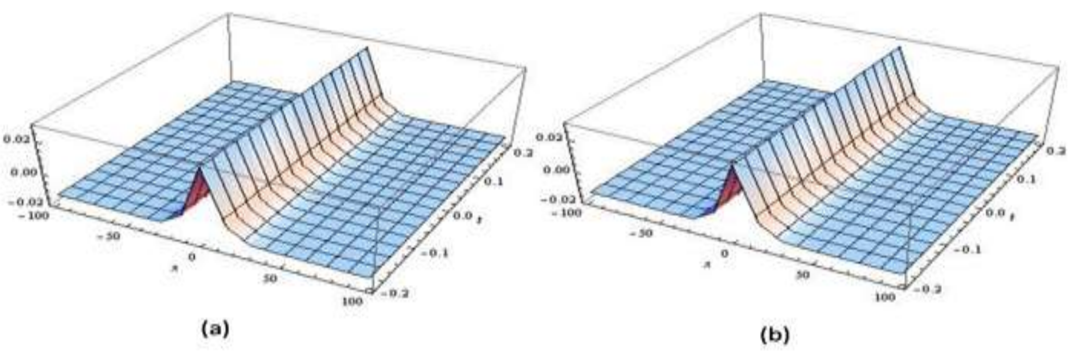


Figure 4. For k = 0.1: (a) exact solution (b) 3-term NIM solution of (30).

IV CONCLUSION

In this paper, New Iterative Method has been successfully used for obtaining the analytic solutions for Lax equations and Sawada-Kotera KdV equations of fifth and seventh orders. NIM solutions are much accurate,as its numerical solutions are closer to the exact solutions than the solutions derived byHPM and ADM. The same has also been validated by the graphical comparison of the NIM solutions with the exact solutions. Further, NIM solutions are computationally efficient as it gives good numerical approximation by considering 2 or 3-terms only. NIM solutions, as obtained in section 3, are shown to be fast and direct as there is no overhead to compute additional terms,such as adomian polynomial in ADM and construction of homotopy in HPM.

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