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ANALYSIS OF FUNCTIONALLY GRADED CYLINDER SUBJECTED TO INTERNAL PRESSURE

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ABSTRACT

In this paper Functionally graded cylindrical shell is analyzed which is subjected to the internal pressure. In most of studies its thermal aspect is studied but here we analyze how it respond to the pressure loading. Using the software COMSOL 4.2 Multiphysics we have found the solutuion in term of stress . The result of FEM simulation (sigmoid law) in COMSOL 4.2 Multiphysics are compared with analytical result in which volume fraction varying by power law for the value of β from -2 to 2. The comparison represents good coincidence between analytical and numerical results and confirms the accuracy of stress and strain solutions presented for cylinderl made up of FGM

Keywords*: *Functionally Graded Cylinder, Sigmoid Gradation Law, FEM Simulation

I. INTRODUCTION

The development of industries going so high to be precise and almost accurate, pipes and pressure vessels have wide spectrum of application in nuclear, chemical, marine and aerospace sectors where operational environment is very dynamic and extreme in terms of thermal and mechanical loadings and their failure can result into hazardous disasters.

Advanced materials such as FGM can be suitable replacement for use in pressure vessel. Ghasemi et al. presented an solution to the FGM cylinder under internal pressure[1]. He proposed and compared the result of a homogeneous cylinder result with the FGM cylinder for different value of β. Mackerle came up with extensive review (1999-2005) of theoretical and empirical analysis of pressure vessel structure/components and pipes and also studied different analysis, applications and components of pressure vessels [2-4]. Kabir devised internally pressurized cylindrical vessels with a load sharing metallic liner in which results exhibited that metallic liner lead to exceptional decrease in on-axis stress both in helical and circular layers [5]. Xia et al. analysed multi-layered filament-wound composite pipes subjected to internal pressure. They introduced an elastic exact solution for stress and deformation distribution of these pipes [6]. Moreover they displayed that gradation put influential affect on stress and displacement distributions.

Hocine et al. also experimentally and analytically investigated a vessel made of carbon/epoxy envelope coated on metallic liner,. They presented an exact solution of stresses and strains on cylindrical section of vessel under mechanical loading and also validated the analytical results by manufacturing and testing some prototype vessels [7]. Baoping et al. presented the reliability-based load and resistance factor design of composite pressure vessel subjected to external hydrostatic pressure and compared it with the deterministic design methods[8].

Despite many advantages of full-composite pressure vessels, they have some limitations such as stress concentration, crack at high temperature and leakage. In the other hand, functionally graded materials (FGMs) have known advantages such as smooth change of physical and mechanical properties. Recently, there has been growing interest in materials deliberately fabricated so that their mechanical properties vary continuously in space on the macroscopic scale.

Using the infinitesimal theory of elasticity, Dai et al. [9] obtained exact solution of FGM cylindrical and spherical vessels subjected to internal pressure and a uniform magnetic field. Shariyat et al. [10] developed analytical and numerical elasto-dynamic solutions for long thick-walled functionally graded cylinders subjected to arbitrary dynamic and shock pressures. Variations of the material properties across the thickness are described according to both polynomial and power law functions. A numerically consistent transfinite element formulation is presented for both functions whereas the exact solution is presented for the power law function. Results obtained for various exponents of the functions of the material properties distributions, various radius ratios, and various dynamic and shock loads.

Literature review of these papers exhibit that most of the research work is limited to analysis FGM vessel it cause to performance improvement. As the FGMs are advanced materials having appealing characteristics and composite material with high strength to weight fraction so their hybrid is a new concept to gain FGMs advantages to improve composite disadvantages. However, a research on stress and strain distribution in hybrid vessels made of FGMs reinforced by laminated composite is a new approach to find new material with prominent properties. FGMs are used with a purpose of reducing the stress concentration, control crack growth and to prevent leakage of fluid in composite pressure vessel.

For these purposes, first analytical solution of thick-walled FGM pressure vessels is considered. Then, numerical study is conducted using the COMSOL Multiphysics 4.2 software. The volume fraction gradation formula is changed as sigmoid gradation law for β -2 t0 2.

II. ANALYTICAL SOLUTION FOR FUNCTIONALLY GRADED CYLINDER

FGMs vessels are thick-wall or thin-wall cylinders that use for different application. The gradation of material for analytic solution is based on power law:-

$$V_c(r) = [(ro - ri)/(ro-r)]^n \qquad (1)$$

Where n is the material grading index and r is the cylinder thickness. The material properties of a S-FGM can be determined by the rule of mixture:

$$E(r) = E_c V_c(r) + E_m[1 - V_c(r)] \qquad (2)$$

Where E_c and E_m are young's modulus the subscripts m and c represents the metallic and constituents, respectively. but simulation of this problem we have changed the gradation law as SIGMOID LAW (S-FGM)

$$V_1 = 1 - \frac{1}{2}\left(\frac{ro-ri/2-r}{(ro-ri)/2}\right)^n \quad for \quad 0 \le r \le (ro - ri)/2 \qquad (3)$$

$$V_2 = \frac{1}{2}\left(\frac{(ro-ri)/2+r}{r/2}\right)^n \quad for \quad -ro - ri/2 \le r \le 0 \qquad (4)$$

The young's modulus of S-FGM can be calculated using rule of mixture:

$$E(r) = V_1(r)E_c + [1 - V_1(r)]E_m \qquad \text{for} \qquad 0 \le r \le \frac{ro-ri}{2} \tag{5}$$

$$E(r) = V_2(r)E_c + [1 - V_2(r)]E_m \qquad \text{for} \qquad 0 \le r \le ro - ri \tag{7}$$

Assuming plane strain and axisymmetry of cylindrical coordinate problem the eqution is expressed in term of :-

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{8}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\sigma_{r\theta}}{r} = 0 \tag{9}$$

Where σ_r and σ_Θ are radial and circumferential stresses and calculate as follows:-

$$\sigma_r = \frac{E(r)}{(1+\upsilon)(1-2\upsilon)}[(1-\upsilon)\varepsilon_r + \upsilon\varepsilon_\theta] \tag{10}$$

$$\sigma_\theta = \frac{E(r)}{(1+\upsilon)(1-2\upsilon)}[\upsilon\varepsilon_r + (1-\upsilon)\varepsilon_\theta] \tag{11}$$

Here ε_r and ε_Θ are calculated as

$$\varepsilon_r = \frac{\partial u}{\partial r} \qquad \varepsilon_\theta = \frac{1}{r}(\frac{\partial v}{\partial \theta} + u) \qquad \gamma_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial r} - \frac{v}{r} \tag{12}$$

III. RESULT AND DISCUSSION

The analysis of thick-walled cylindrical vessel is performed using the COMSOL Multiphysics 4.2 software. Geomtric, Material and also boundary conditions are given in Table 1. To eliminate the effect of boundary condition on stress, the stress results are read far from the vessel edge. The element to the model in simulation used is triangular with fine meshing with total degree of freedom 2064. For the simulation perform by 2D - axissymmetric physics with stationary solution is selected. This model is a precise approximation method to simulate the FGM vessel.

Table 1

υ	E_0(GPa)	R(mm)	a(mm)	P(MPa)
0.3	72	30	50	7

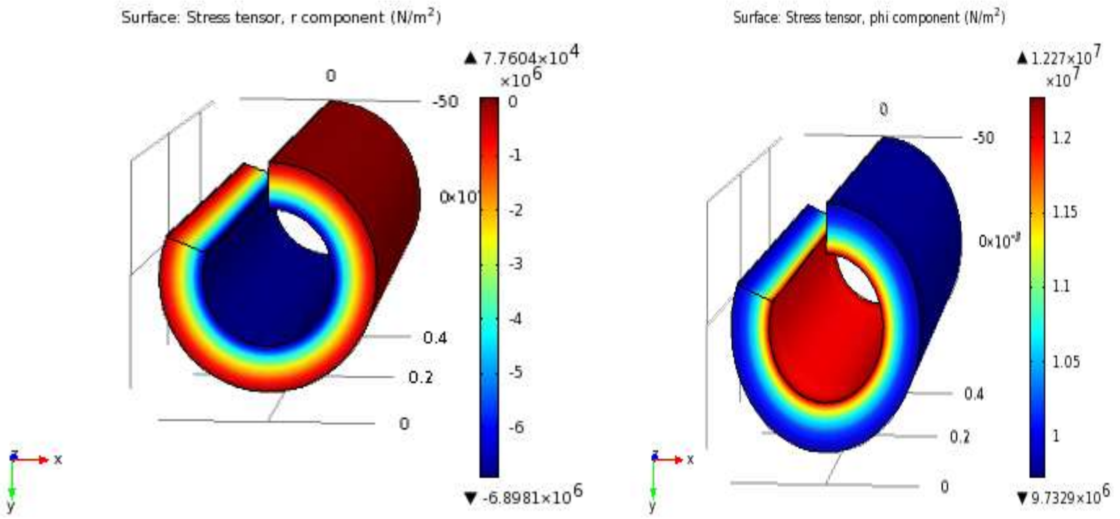

Fig. 1 Stress distribution of FGM vessel modeling under internal pressure.

Table.2 Radial and circumferential stresses of FGM cylinder for β=1.

R(mm)	Analytical[11,12]]		FEM		Error(%)	
	σ_r(MPa)	σ_Θ(MPa)	σ_r(MPa)	σ_Θ(MPa)	σ_r(MPa)	σ_Θ(MPa)
30.0	-7.00	11.17	-6.90	12.27	0.24	6
32.5	-5.61	10.88	-5.59	11.34	.3	4.2
35.0	-4.44	10.67	-4.42	10.92	.4	2.34
37.5	-3.44	10.51	-3.43	10.62	.2	1.04
40.0	-2.57	10.40	-2.57	10.34	1.9	5
42.5	-1.81	10.33	-1.75	10.16	3.3	1.6
45.0	-1.14	10.28	-1.10	9.99	3.5	2.6
47.5	-.54	10.25	-.52	9.84	3.7	2.6
50.0	0.00	10.24	00	9.74	0	4.1

In Table 1. υ , E , a , R and P are Poission's ratio ,elastic constant , inner radius , outer radius and internal pressure of the vessel , respectively . Modeling and simulation is done for β=1. Distribution of stress in radial and circumferential direction by FEM modeling of FGM vessel is shown in Fig.1.

Radial and circumferential stress from analytical solution and the FEM and the difference between them for β=1 is given in the Table. 2. In this table the maximum difference is 3.3% and 6% respectively. Maximum difference is towards inner radius and minimum is near outer radius as Saint Venant principal effected on FEM. Hence the precise analytical and FEM Result has good agreement and error always less then 6%.

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