FUZZY δ-PERFECTLY SUPER CONTINUOUS MAPPING

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ABSTRACT

In this paper we studied and introduced a new class of mapping called fuzzy δ-perfectly super continuous mapping and explore some of its properties in fuzzy topological spaces.

Key Words: (Fuzzy Super Continuous Mapping, Fuzzy D-Super Super Continuous Mapping, (Fuzzy Almost) Strongly δ-Super Continuous Mapping, Fuzzy δ-Partition Topology, Fuzzy Clustered Space, Fuzzy Saturated Space.

I PRELIMINARIES

Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha \in \Lambda\}$ of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta (y) = \beta$ for y=x and x(y) =0 for $y \neq x$, $\beta \in [0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x \in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1

A subset A of a fuzzy topological space (X,τ) is called

[I]. Fuzzy Super closure scl(A)={x∈X:cl(U)∩A≠ϕ}

[II]. Fuzzy Super interior sint(A) ={x∈X:cl(U)≤A≠ϕ}

[III]. Fuzzy super closed if scl(A) ≤ A.

[IV]. Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Let X be a fuzzy topological space and A be a fuzzy set of X. The fuzzy interior (resp .fuzzy closure) of a fuzzy set A in X will be denoted by int(A) (resp.cl(A)).

Definition 1.2: A subset A of a fuzzy topological space X is called

I. fuzzy pre super open set if A ≤ int(cl (A)) and a pre super closed set if cl(int (A))≤ A.
II. fuzzy semi super open set if A ≤clint (A) and a semi super closed set if int(cl (A))≤A.
III. fuzzy regular super open set if A = int(cl (A)) and a regular super closed set if A = cl(int(A)).
IV. fuzzy π- super open set if A is a finite union of regular super open sets.
V. fuzzy regular semi super open[4]if there is a regular super open U such that U ≤A ≤ cl(U).

Definition1.3: A subset A of (X,τ) is called;

I. Fuzzy generalized super closed set (briefly, fuzzy g-super closed) if cl (A)≤ U whenever A ≤ U and U is super open in X.
II. Fuzzy regular generalized super closed set (briefly, fuzzy rg-super closed) if cl (A)≤U whenever A ≤ U and U is fuzzy regular super open in X.
III. Fuzzy generalized pre regular super closed set (briefly, fuzzy gpr-super closed) if pcl(A)≤U whenever A ≤ U and U is fuzzy regular super open in X.
IV. Fuzzy weakly generalized super closed set (briefly, fuzzy wg- super closed) if cl(int (A))≤U whenever A≤ U and U is fuzzy super open in X .
V. Fuzzy π-generalized super closed set (briefly, fuzzy πg-super closed) if cl(A)≤U whenever A≤U and U is fuzzy π- super open in X.
VI. Fuzzy weakly super closed set (briefly, fuzzy w-super closed) if cl(A) ≤U whenever A≤U and U is fuzzy semi super open in X.
VII. Fuzzy regular weakly generalized super closed set (briefly, fuzzy rwg- super closed) if cl(int(A))≤U whenever A ≤U and U is fuzzy regular super open in X. Fuzzy rw-super closed if cl(A)≤U whenever A≤U and U is fuzzy regular semi super open.
VIII. Fuzzy g*-super closed if cl(A)≤U whenever A≤U and U is fuzzy w- super open.
IX. Fuzzy rg-super closed if cl(A)≤U whenever A≤U and U is fuzzy rw- super open.

II PRELIMINARIES AND BASIC DEFINITIONS

Definitions 2.1.: A function f : X → Y from a fuzzy topological space X in to a fuzzy topological space Y is said to be

[I]. Fuzzy Strongly super continuous if f(cl(A))≤ f(A) for each subset A of X.

[II]. Fuzzy perfectly super continuous if f^{-1}(V) is fuzzy clopen in X for every fuzzy open set V ≤ Y .

[III]. Fuzzy Almost perfectly super continuous (fuzzy regular set connected if f^{-1}(V) is fuzzy cl- open for every regular fuzzy open set V in Y .

[IV]. Fuzzy cl-super super continuous (fuzzy clopen super continuous) if for each x ∈X and each fuzzy open set V containing f(x) there is a fuzzy clopen set U containing x such that f(U) ≤ V .

[V]. Fuzzy almost cl-super super continuous (fuzzy almost clopen) if for each x∈X and for each fuzzy regular open set V containing f(x) there is a fuzzy clopen set U containing x such that f(U) ≤V .

[VI]. Fuzzy almost strongly θ-super continuous if for each x ∈X and for each (fuzzy regular) open set V containing f(x), there exists a fuzzy open set U containing x such that f(cl(U))≤ V .

[VII]. Fuzzy super continuous if for each x ∈X and for each fuzzy open set V containing f(x), there exists a fuzzy regular open set U containing x such that f(U) ≤V .

[VIII]. Fuzzy almost z-super super continuous if for each x ∈X and each fuzzy regular open set V containing f(x), there exists a co zero set U containing x such that f(U) ≤V .

[IX]. fuzzy almost D_δ-super super continuous if for each x ∈X and each fuzzy regular open set V containing f(x), there exists a fuzzy regular F_σ-set U containing x such that f(U) ≤V .

[X]. fuzzy δ-super continuous if for each x ∈X and for each fuzzy regular open set V containing f(x), there exists a fuzzy regular open set U containing x such that f(U) ≤V .

[XI]. fuzzy almost super continuous if for each x ∈X and for each fuzzy regular open set V containing f(x), there exists a fuzzy open set U containing x such that f(U) ≤V .

Definition 2.2.: A set G is said to be fuzzy δ- open if for each x ∈G, there exists a fuzzy regular open set H such that x ∈H ≤ G, or equivalently, G is expressible as an arbitrary union of fuzzy regular open sets. The complement of a fuzzy δ- open set will be referred to as a fuzzy δ- closed set.

Definition 2.3.: Let X be a fuzzy topological space and let A≤X. A point x ∈X is called a fuzzy δ-adherent point(fuzzy cl-adherent point) of A if every fuzzy regular open set (fuzzy cl open set) containing x has non-empty intersection with A. Let A_δ ($[A]_{cl}$) denote the set of all fuzzyδ-adherent points (fuzzy cl-adherent points)of A. The set A is fuzzyδ- closed (fuzzy cl- closed) if and only if A = A_δ ($[A]_{cl}$ = A).

Definitions 2.4.: A space X is said to be endowed with a

[I]. Fuzzy Partition topology if every fuzzy open set in X is fuzzy closed.

[II]. Fuzzy δ-Partition topology if every fuzzy δ- open set in X is fuzzy closed or equivalently every fuzzy δ-closed set in X is fuzzy open.

III FUZZY δ- PERFECTLY SUPER CONTINUOUS FUNCTIONS

A function f : X → Y from a fuzzy topological space X into a fuzzy topological space Y is said to be fuzzyδ-perfectly super continuous if for each fuzzyδ- open set V in Y ,f^{-1}(V) is a fuzzy clopen set in X.

Theorem 3.1: For a function f: X → Y from a fuzzy topological space X into a fuzzy topological space Y, the following statements are equivalent:

(a) f is fuzzy δ- perfectly super continuous .

(b) f^{-1}(F) is fuzzy clopen in X for every fuzzyδ-closed set F in Y .

(e) $f(A_{cl}) \leq [f(A)]_{\delta}$ for every subset A≤X.

Proof: Easy.

Definition 3.2.: Let Y be a fuzzy subspace of a fuzzy space Z. Then Y is said to be fuzzyδ-embedded in Z if every fuzzyδ- open set in Y is the restriction of a fuzzyδ- open set-in Z with Y or equivalently every fuzzyδ- closed set in Y is the restriction of a fuzzyδ- closed set in Z with Y.

Theorem 3.3.: Let f : X →Y be a fuzzy δ- perfectly super continuous function. If f(X) is fuzzy δ-embedded in Y , then the surjection f : X → f(X) is fuzzyδ- perfectly super continuous .

Proof. Let V be a fuzzyδ- open set in f(X). Then there exists a fuzzyδ- open set W in Y such that V = W∩f(X). It follows that $f^{-1}(V) = f^{-1}(W) \cap X = f^{-1}(W)$.

Theorem 3.4.: If f: X → Y is fuzzyδ- perfectly super continuous function and g : Y → Z is a fuzzy δ-super continuous function, then gof is fuzzy δ- perfectly super continuous . In particular, the composition of two fuzzy δ- perfectly super continuous functions is fuzzyδ- perfectly super continuous.

Proof.: Let W be a fuzzy δ- open set in Z. Since g is fuzzyδ-super continuous, g^{-1}(W) is fuzzy δ- open in Y (see [14]). In view of fuzzyδ-perfect continuity of f, $f^{-1}(g^{-1}(W))$ is fuzzy clopen in X. Since $(gof)^{-1}(W) = f^{-1}(g^{-1}(W))$, gof is fuzzyδ- perfectly super continuous . It is routine to verify that fuzzyδ-perfect continuity is invariant under the restriction of domain

Theorem 3.5. : Let f : X → Y be a fuzzy function and let Q = $\{X_\alpha : \alpha \in \Lambda\}$be a locally finite fuzzy clopen cover of X. For each α∈ Λ, let $f_\alpha = f|X_\alpha$ denote the restriction map. Then f is fuzzyδ- perfectly super continuous if and only if each $f_\alpha$is fuzzyδ- perfectly super continuous.

Proof. Necessity is immediate since fuzzyδ-perfect continuity is invariant under restriction of domain. To prove sufficiency, let V be a fuzzy δ- open set in Y. Then

$f^{-1}(V) = \cup (f \mid X_\alpha)^{-1}(V) = \cup (f^{-1}(V) \cap X_\alpha)$. α∈Λα∈Λ

Since each $f^{-1}(V) \cap X_\alpha$is fuzzy clopen in X_α and hence in X. Thus f^{-1}(V) is fuzzy open being the union of fuzzy clopen sets. Moreover, since the collection Q is locally finite, the collection $\{f^{-1}(V) \cap X_\alpha : \alpha \in \Lambda\}$ is a locally finite collection of fuzzy clopen sets. Since the union of a locally finite collection of fuzzy closed sets is fuzzy closed, f^{-1}(V) is also fuzzy closed and hence fuzzy clopen.

Theorem 3.6. :If f : X → Y is a fuzzyδ- perfectly super continuous surjection which maps fuzzy clopen sets to fuzzy closed sets (fuzzy open sets). Then Y is endowed with a fuzzyδ-partition topology. Further, if in addition f is a bijection which maps fuzzy δ- open (fuzzy δ- closed) sets to fuzzy δ- open (fuzzy δ- closed) sets, then X is also equipped with a fuzzy δ-partition topology.

Proof. Suppose f maps fuzzy clopen sets to fuzzy closed (fuzzy open) sets. Let V be a fuzzy δ- open (fuzzy δ-closed) set in Y. Since f is fuzzy δ- perfectly super continuous, $f^{-1}(V)$ is a fuzzy clopen set-in X. Again, since f is a surjection which maps fuzzy clopen sets to fuzzy closed (fuzzy open)sets, the set $f(f^{-1}(V)) = V$ is fuzzy closed (fuzzy open) in Y and hence fuzzy clopen. Thus Y is endowed with a fuzzyδ-partition topology.

To prove the last part of the theorem assume that f is a bijection which maps fuzzyδ- open (fuzzy δ-closed) sets to fuzzyδ-open (fuzzy δ-closed) sets. To show that X possesses a fuzzyδ-partition topology; let A be a fuzzy δ- open (fuzzy δ-closed) set in X. Then f(A) is a fuzzy δ- open (fuzzyδ- closed) set in Y . Since f is a fuzzy δ-perfectly super continuous bijection, $f^{-1}(f(A)) = A$ is a fuzzy clopen set in X. This proves that X is endowed with a fuzzy δ-partition topology.

Theorem 3.7: Let $f : X \to Y$ be a fuzzy function and $g : X \to X\times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, be the graph function. If g is δ-fuzzy perfectly super continuous, then so is f and the fuzzy space X possesses a fuzzy δ-partition topology. Further, if f is fuzzy δ- perfectly super continuous and X is endowed with a fuzzy δ-partition topology, then g is fuzzy δ- perfectly super continuous.

Proof. : Suppose that the fuzzy graph function $g : X \to X \times Y$ is fuzzy δ- perfectly super continuous . Now, it is easily verified that the fuzzy projection map $p_y : X\times Y \to Y$ is fuzzy δ- super continuous so in view of Theorem 4.4 the function $f = p_y \circ g$ is fuzzy δ- perfectly super continuous . To prove that X is fuzzyδ-partition topology, let Ube a fuzzy δ- open set in X. Then U×Y is a fuzzy δ- open in X×Y. Since g is fuzzy δ- perfectly super continuous, $g^{-1}(U \times Y) = U$ is fuzzy clopen in X and so the fuzzy topology of X is a fuzzy δ-partition topology.

Conversely, suppose that f is fuzzy δ- perfectly super continuous and let X be endowed with a fuzzy δ-partition topology. To show that g is fuzzy δ- perfectly super continuous let W be fuzzy δ- open set in X × Y. Suppose W = $\{U_\alpha \times V_\alpha : U_\alpha$ is fuzzy regular open in X and V_α is fuzzy regular open in Y $\}$. Then $p_y(W) = \cup\ V_\alpha = V$ is fuzzy δ-open in Y . $\alpha \in \Lambda$ Since f is fuzzy δ- perfectly super continuous, $f^{-1}(V)$ is fuzzy clopen in X. Now, since $g^{-1}(W) = f^{-1}(p_y(W)) = f^{-1}(V)$, g is fuzzyδ-fuzzy perfectly super continuous .

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