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SYMMETRIC BI-DERIVATIONS ON SEMIPRIME RINGS

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ABSTRACT

Let R be a ring . A symmetric biadditive mapping $D(.,.): R \times R \to R$ is called a symmetric biderivation, if for any fixed y in R, the mapping x $\mapsto$ D(x,y) is a derivation . The purpose of this paper is to extend a important result of Deng for n- centralizing traces of symmetric biderivations concerning semiprime rings.

Keywords : Prime Rings, Semiprime Ring , Torsion Free Rings, Derivation, Symmetric Biderivatios

Mathematics Subject Classification: 16R50, 16W25, 16N60

I INTRODUCTION

Throughout the paper R will denote a ring and Z(R), the centre of R. A ring is said to be prime if aRb=0 implies that either a = 0 or b = 0. We shall write [x,y] = xy - yx and use the identities [xy,z] = [x,z]y +x[y,z] and [x,yz]=[x,y]z +y[x,z] . An additive mapping $d: R \to R$ is called a derivation if d(xy)=d(x)y + xd(y) for all x,y in R. A symmetric biadditive mapping $D(.,.): R \times R \to R$ is called a symmetric biderivation, if for any fixed y in R, the mapping x $\mapsto$ D(x,y) is a derivation. A mapping $f: R \to R$ defined by f(x)= D(x,x), where $D(.,.): R \times R \to R$ is a symmetric mapping is called a trace of D. It is obvious that in case $D(.,.): R \times R \to R$ is a symmetric mapping which is also biadditive, the trace f of D satisfies the relation f(x + y) = f(x) + f(y) + 2D(x,y) for all x,y in R. Let σ and τ be automorphisms on R. A symmetric biadditive mapping $D(.,.): R \times R \to R$ is called a symmetric (σ, τ) biderivation, if D(xy,z)=D(x,z)σ (y)+ τ (x)D(y,z) saitisfied for all x, y, z in R.

In 1980, Gy, Maksa [7] introduced the concept of a symmetric biderivation on a ring R (see also [8], where an example can be seen). It was shown in [8] that symmetric biderivations are related to general solution of some functional equations. Some results on a symmetric biderivation in prime and semiprime rings can be found in [13] and [14]. The notion of additive commuting mappings is closely connected with the notion of biderivations. Every commuting additive mapping $f: R \rightarrow R$ gives rise to a biderivation on R. Linearizing [f (x), x] =0, for all x in R, we get [f (x), yl = [x, f (y)], for all x, y in R and hence we note that the mapping (x, y) $\rightarrow$ [f (x), y] is a biderivation on R (moreover, all derivations appearing are inner).

There has been considerable interest for commuting, centralizing and related mappings in prime (semiprime) rings (see [1, [2], [4], [9], [10], [11], [15] etc., where further references can be found). The most fundamental result in the theory of centralizing mappings is a theorem of Posner [12], which states that if a derivation d of a non-commutative ring R satisfies [d(x), x] $\in$ Z(R),for all x in R, then d =0. A number of authors have extended Posner's theorem in several directions.

Motivated by the definition of centralizing and commuting mappings, Deng and Bell [6] defined n-centralizing and n-commuting mappings as: Let n be an arbitrary positive integer and S be a nonempty subset of a ring R. A mapping $f: R \rightarrow R$ is said to be n-centralizing (resp. n-commuting) on S if $[x^n, f(x)] \in Z(R)$ (resp.$[x^n, f(x)] = 0$) holds for all x $\in$ S.

In the same paper they proved that if U is a nonzero left ideal of an n!-torsion free semiprime ring R and d : R$\rightarrow$ R is a derivation which is n-centralizing on U, then d must be n-commuting on U .

Bell and Martindale [1] proved that if a semiprime ring R admits a derivation d which is nonzero on a nonzero left ideal L of R and centralizing on L ,then R must contain a nonzero central ideal. Deng and Bell [6] generalized this result for n-centralizing mappings. In the present paper, we establish the same result for symmetric biderivation on semiprime ring.

II MAIN RESULT

Theorem 2.1. Suppose that n > 1 is a fixed positive integer. Let R be a 2,3and (2^n - 1)-torsion free semiprime ring and I be a nonzero ideal of R. Suppose there exists a symmetric biderivation $D(.,.): R \times R \rightarrow R$ such that the mapping $f: R \rightarrow R$ is n-centralizing on I, where f stands for the trace of D. Then f is n-commuting on I.

Proof : By the assumption we have

$$[x^n, f(x)] \in Z(R), \quad \text{for all x in I} \quad (2.1)$$

Replacing x by x^2 in (2.1), we have

$$[x^{2n}, f(x^2)] = [x^{2n}, x^2 f(x) + 2xf(x)x + f(x)x^2] \in Z(R)$$

That is

$$x^2[x^n, f(x)]x^n + x^{n+2}[x^n, f(x)] + 2x^{n+1}[x^n, f(x)]x + 2x[x^n, f(x)]x^{n+1} + x^n[x^n, f(x)]x^2 +$$
$$[x^n . f(x)]x^{n+2} \in Z(R)$$

The above relation reduces to $8x^{n+2}[x^n, f(x)] \in Z(R)$ for all x in I using (2.1). Since R is a 2-torsion free, we have

$$[x^{n+2}, f(x)][x^n, f(x)] = 0, \qquad \text{for all x in I} \qquad (2.2)$$

Now linearzing (2.1), we obtain

$$[x^n, f(y)] + [x^{n-1}y + x^{n-2}yx + ... + yx^{n-1}, f(x)] + [y^n, f(x)] +$$
$$[y^{n-1}x + y^{n-2}xy + ... + xy^{n-1}, f(y)] + [x^n, 2D(x, y)] + [x^{n-1}y + x^{n-2}yx + ... + yx^{n-1}, 2D(x, y)]$$
$$+ [y^n, 2D(x, y)] + [y^{n-1}x + y^{n-2}xy + ... + yx^{n-1}, 2D(x, y)] = 0$$

for all x,y in I. (2.3)

Replacing x by –x in (2.3) and comparing with (2.3), we have

$$[x^{n-1}y + x^{n-2}yx + ... + yx^{n-1}, f(x)] + [y^{n-1}x + y^{n-2}xy + ... + xy^{n-1}, f(y)] +$$
$$2[x^n, D(x, y)] + 2[y^n, D(x, y)] \in Z(R)$$

For all x,y in I. (2.4)

Substituting 2x in place of x in (2.4), we get

$$2^{n+1}[x^{n-1}y + x^{n-2}yx + ... + yx^{n-1}, f(x)] + 2[y^{n-1}x + y^{n-2}xy + ... + xy^{n-1}, f(y)] +$$
$$2^{n+2}[x^n, D(x, y)] + 2^2[y^n, D(x, y)] \in Z(R)$$

For all x,y in I. (2.5)

Using 2-torsion freeness in (2.5) and comparing with (2.4), we obtain

$$(2^n - 1)[x^{n-1}y + x^{n-2}yx + ... + yx^{n-1}, f(x)] + (2^{n+1} - 2)[x^n, D(x, y)] \in Z(R)$$

Since R is a (2^n-1)-torsion free, the above relation reduces to

$$[x^{n-1}y + x^{n-2}yx + ... + yx^{n-1}, f(x)] + 2[x^n, D(x, y)] \in Z(R), \text{ for all x,y in I} \qquad (2.6)$$

Substituting x^3 in place of y in (2.6), we have

$n[x^{n+2}, f(x)] + 2x^2[x^n, f(x)] + 2x[x^n, f(x)]x + 2[x^n, f(x)]x^2 \in Z(R)$, for all x in I (2.7)

Using (2.1), we get

$n[x^{n+2}, f(x)] + 6x^2[x^n, f(x)] \in Z(R)$, for all x in I (2.8)

Since $[x^n, f(x)] \in Z(R)$, it follows that $n[x^{n+2}, f(x)][x^n, f(x)] + 6x^2[x^n, f(x)]^2 \in Z(R)$. Application of (2.2) gives that $6x^2[x^n, f(x)]^2 \in Z(R)$ for all x in I. Also 2 and 3 torsion freeness gives $x^2[x^n, f(x)]^2 \in Z(R)$ for all x in I. Since $(x[x^n, f(x)])^{2n} = x^{2n}[x^n, f(x)]^{2n} \in Z(R)$, it follows that $[x^{2n}[x^n, f(x)]^{2n}, f(x)] = 0$, for all x in I. By hypothesis $[x^n, f(x)] \in Z(R)$, implies that $[x^{2n}, f(x)][x^n, f(x)]^{2n} = 0$ for all x in I. Thus we have $2x^n[x^n, f(x)]^{2n+1} = 0$, for all x in I and 2-torsion freeness of R yields that $x^n[x^n, f(x)]^{2n+1} = 0$, for all x in I. Hence $[x^n, f(x)][x^n, f(x)]^{2n+1} = [x^n, f(x)]^{2n+2} = 0$, for all x in I. Since centre of semi-prime ring contains no nonzero nilpotent elements, we have $[x^n, f(x)] = 0$, for all x in I, completes the proof .

Conjecture 2.2. Suppose that n > 1 is a fixed positive integer. Let R be a 2,3 and (2^n - 1)-torsion free semiprime ring and I be a nonzero ideal of R. Suppose there exists a symmetric (σ, τ) biderivation $D(.,.) : R \times R \to R$ such that the mapping $f : R \to R$ is n-centralizing on I, where f stands for the trace of D and σ, τ are automorphisms . Then f is n-commuting on I.

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