Interaction of a Normal Shock with a Yawed Wedge Moving at Supersonic Speed

R.S. Srivastava[1], Sanjay Srivastava[2]

[1]*Formerly of Defence Science Centre, New Delhi, (India)*

[2]*Bechtel (India)*

ABSTRACT

In this paper the interaction of a normal shock with a yawed wedge moving at supersonic speed has been considered. The problem has been considered earlier for Mach number of the shock wave fixed and varying the Mach number of the moving wedge. Here we have carried out the calculation for various Mach numbers of the moving wedge keeping the Mach number of the shock wave fixed.

***Keywords: Diffraction, Yawed wedge, Vorticity distribution, Mach reflection*.**

I. INTRODUCTION

Lighthill (1949) considered the diffraction of a normal shock wave past a small bend of small angle δ. Chester (1954) extended the work of Lighthill (1949) to the case of yawed wedge i.e. to the case when there an angle between the shock front and leading edge of the wedge. Smyrl (1963) considered the interaction of normal shock with a wedge of small angle moving at supersonic speed in opposite direction both for unyawed and yawed cases following Lighthill (1949) and Chester (1954). Srivastava (2005) gave the vorticity distribution of a particle over the diffracted shock for the case of unyawed wedge. In (2009), Srivastava obtained the vorticity distribution over the diffracted shock for the yawed wedge case. The Mach number of the shock wave and Mach number of the wedge and wedge angle β are relevant data for the consideration of the problem. In (2009) we fixed the Mach number of the shock wave and varied the Mach number of the wedge for varying β's. In the present paper we have solved the problem of (2009) for fixed Mach number of the wedge and varying Mach number of shock wave for varying β's. Srivastava (1994) book may be referred for detail reference.

II. MATHEMATICAL FORMULATION

The velocity of the shock wave is U and Mach number of the shock wave is $M = U/C_0$. The leading edge of the wedge is moving with supersonic speed W and its Mach number is W/C_0. C_0 is the speed of sound ahead of shock wave.

There is an angle β between shock wave and leading edge of the wedge. We see that the component of velocity U along the leading edge is $U\operatorname{cosec}\beta$ and the component of velocity W along the shock front is $W\operatorname{cosec}\beta$.

The resultant of these two velocities is $\vec{V}_0$ whose magnitude is given by

$$V_0 = \left[\frac{U^2}{\sin^2\beta} + \frac{W^2}{\sin^2\beta} + 2\left(\frac{U}{\sin\beta}\cdot\frac{W}{\sin\beta}\right)\cos\beta\right]^{1/2}$$

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$$=\left[U^2+W^2+2UW\cos\beta\right]^{1/2}\operatorname{cosec}\beta \qquad -(1)$$

By imposition of a velocity $\vec{V}_0$ on the entire system, the point of intersection of shock front and the leading edge comes to rest and a velocity $\vec{V}_0$ appears in the region behind the shock (Srivastava 2009). This is the definition of $\vec{V}_0$. $\vec{V}_0$ intersect with the shock front at an angle β'

$$\beta'=\sin^{-1}\left(U/V_0\right)=\sin^{-1}\frac{\sin\beta}{\left(1+\frac{M'^2}{M^2}+\frac{2M'}{M}\cos\beta\right)^{1/2}} \qquad -(2)$$

where specific heats γ is assumed to be 1.4. The uniform flow behind the shock has now the velocity $\vec{V}_1''=\vec{V}_1+\vec{V}_0$, the direction of $\vec{V}_1''$ makes an angle μ with the shock front, where

$$\tan\mu=\frac{(U-V_1)\sin\beta}{W+U\cos\beta}=\frac{(M^2+5)\sin\beta}{6M(M'+M\cos\beta)} \qquad -(3)$$

$$V_1''^2=(U-V_1)^2+\left\{\frac{W+U\cos\beta}{\sin\beta}\right\}^2 \qquad -(4)$$

V_1 is the uniform flow behind the shock before interaction.

We assume supersonic flow behind the shock wave and hence the perturbations introduced by the presence of the wedge are confined to the region bounded by the shock front, wedge surface and Mach cone with vertex 0 at the junction of shock wedge leading edge. The point 0 is taken as origin with the Z-axis in the direction of $\vec{V}_1''$ and the Mach Cone with semi angle α is drawn on the axis with 0 as vertex, where

$$\sin\alpha=\frac{C_1}{V_1''}=\frac{C_1\sin\mu}{(U-V_1)}=\frac{6M'\sin\mu}{(M^2+5)} \qquad -(5)$$

C_1 is the velocity of sound behind the shock.

Let (X,Y,Z) be the rectangular coordinates with the origin at the point 0 and Z-axis along the axis of the Mach cone (Srivastava 2009). In the non-uniform region we use the conservation equations

$$\begin{aligned}&\nabla(\rho\vec{V})=0\\&(\vec{V}.\nabla)\vec{V}=-\frac{1}{\rho}(\nabla p)\\&(\vec{V}.\nabla)(p\rho^{-\gamma})=0\end{aligned} \qquad -(6)$$

$\vec{V}$ is the velocity, p is the pressure and ρ is the density, γ is the ratio of specific heats.

We further take

$$\begin{aligned}p&=p_1+\in p^{(1)}(X,Y,Z)+\in^2 p^2(X,Y,Z)+\cdots\\\rho&=\rho_1+\in\rho^{(1)}(X,Y,Z)+\in^2\rho^2(X,Y,Z)+\cdots\\\vec{V}&=\vec{V}_1''+\in\vec{V}^1(X,Y,Z)+\in^2\vec{V}^2(X,Y,Z)+\cdots\end{aligned} \qquad -(7)$$

$\vec{V}_1''$ is the uniform flow behind the shock after interaction, p_1, ρ_1 are the pressure and density behind the shock.

We now introduce the following transformations

$$x = \frac{X}{Z\tan\alpha}, \quad y = \frac{Y}{Z\tan\alpha}$$

$$p' = \frac{p^{(1)}}{\rho_1 C_1^2}, \quad \rho' = \frac{\rho^{(1)}}{\rho_1} \qquad - (8)$$

$$\vec{V}'' = (u'\cos\alpha, \quad v'\cos\alpha, \ -w'\sin\alpha) - (9)$$

Using equations (6), (7), (8) we obtain a single second order differential equations in p'.

$$\nabla^2 p' = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p'}{\partial x} + y\frac{\partial p'}{\partial y}\right) \qquad - (9)$$

The characteristics of the differential equation (9) are the tangents to the unit circle $x^2 + y^2 = 1$.

In the unyawed case, the unit circle lies in the (*x*, *y*) plane which is the cross-section perpendicular to the axis of the cylinder extending upto infinity. In the yawed case the unit circle which lies in the (x, y) plane could be considered perpendicular to the axis of the Mach cone. The section is shown by Srivastava (2009). The expressions for the coordinates on the section are available in the paper by Srivastava (2009) and also in paper by Smyrl (1963).

III. VORTICITY DISTRIBUTION

The Vorticity distribution ζ is given by

$$\zeta = \frac{1}{2}\left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right) \qquad - (10)$$

Following Smyrl (1963) we have

$$u' = \frac{5}{6}\frac{\cos\mu}{\cos\alpha}\cdot\left\{\frac{\cos^2\mu}{\cos\alpha}\frac{\left(M^2 + M'^2 + 2MM'\cos\beta\right)^{1/2}}{M' + M\cos\beta}\right. .$$

$$\cdot\left[\cos(\beta' + \mu) + \frac{1}{M^2}\cos(\beta' - \mu)\right]\left(f(y) - yf'(y)\right)$$

$$\left. + \frac{C_0/C_1}{\left(M'^2 - 1\right)^{1/2}}\left[\frac{M'\cos\beta}{M^2} + \left(\frac{4}{5}\cos\mu - \cos\beta\right)M' - \frac{2}{5}\frac{M'^2}{M}\right]\right\} \qquad - (11)$$

The position of the shock front is given by

$$x = x_0 + \in f(y) \qquad - (12)$$

where $\in$ *f(y)* is small, $\in$ being the angle of bend and *f(y)* is function of *y*. In (11) *f(y)* has this meaning we have

$$v' = -M_1 f'(y)\frac{\cos\mu}{\cos\alpha} + \frac{C_0}{C_1}\frac{M'}{\cos\alpha} \qquad - (13)$$

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We have form (13)

$$\frac{\partial v'}{\partial x} = 0 \qquad - (14)$$

and from (11)

$$\frac{\partial u'}{\partial y} = A - y f''(y) \qquad - (15)$$

where A is a constant and known from equation (11)

If R is the curvature of the diffracted shock, we obtain from the relation (12)

$$R = -f''(y) \in \qquad - (16)$$

Combining the equation (15) and (16) we have

$$\zeta = -Ay\left(\frac{R}{\in}\right) = -A\frac{y}{y_0} \cdot y_0\left(\frac{R}{\in}\right) \qquad - (17)$$

where following Srivastava (2005) and Srivastava (2002)

$$\frac{R}{\in} = \frac{1}{M_1} \cdot \frac{B'}{y_0^2}(\xi+1)^2 \frac{\cos\alpha}{\cos\mu} \cdot \frac{[K_1(\xi-\xi_1)+K_2(\xi-\xi_2)+K_3(\xi-\xi_1)(\xi-\xi_2)] \cdot (\gamma_1+\gamma_2)}{[\gamma_2^2+\xi-1][\gamma_1^2+\xi-1](\xi+1)^{1/2}(\xi-\xi_1)(\xi-\xi_2)} \qquad - (18)$$

If we substitute $R/\in$ from (18) in (17), then the vorticity ζ is known.

In (18) all the factors are constant and functions of M', M and β except ξ which runs from 1 to ∞ on the diffracted shock.

In (17), the factor y/y_0 is given by

$$\frac{y}{y_0} = \left(\frac{\xi-1}{\xi+1}\right)^{1/2} \qquad - (19)$$

$\xi = 1$ is the point where the shock intersects the wedge surface and $\xi \to \infty$ is the point where the shock intersects the unit circle. From (19), therefore we see that on the diffracted shock y/y_0 runs from 0 to 1.

IV. NUMERICAL RESULTS

In the present calculation M' is fixed and M is varied. The calculations have been carried out for the combinations $M' = 2,\ M = 1.5,\ M' = 2,\ M = 2,\ M' = 4,\ M = 2$ and β taking values 0.5, 1.25, 1.5 radians in each case. In each set of calculations the vorticity remains zero at the intersection of shock wave and wedge surface $\left(y/y_0 = 0\right)$ and positive and negative values over diffracted shock wave depending on the choice of the parameters and finally becomes zero at the intersection of Mach circle and shock wave $\left(y/y_0 = 1\right)$. The calculations are shown in the following tables (Table-1 to Table-9)

Table-1 $M' = 2,\ M = 1.5,\ \beta = 0.5$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	0.12	0.34	2.06	7.55	0

Table-2 $M' = 2,\ M = 1.5,\ \beta = 1.25$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	– 0.73	– 1.16	– 0.87	2.90	0

Table-3 $M' = 2,\ M = 1.5,\ \beta = 1.5$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	– 0.82	– 1.01	– 1.22	4.35	0

Table-4 $M' = 2,\ M = 2,\ \beta = 0.5$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	0.31	0.92	2.30	7.36	0

Table-5 $M' = 2,\ M = 2,\ \beta = 1.25$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	– 0.22	0	0.97	4.3	0

Table-6 $M' = 2,\ M = 2,\ \beta = 1.5$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	– 0.38	– 0.57	0.43	2.34	0

Table-7 $M' = 2,\ M = 4,\ \beta = 0.5$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	– 0.34	0	0.07	0.57	0

Table-8 $M' = 2,\ M = 4,\ \beta = 1.25$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	– 0.22	– 0.10	0.16	1.34	0

Table-9 $M' = 2,\ M = 4,\ \beta = 1.5$

y/y_0	0	0.2	0.4	0.6	0.8	1
ζ	0	– 0.05	0	0.45	1.46	0

V. MACH REFLECTION

The triple point angle χ in Mach reflection is given by the mathematical relation

$$\tan(\chi + \in \cos\beta) = \frac{\sqrt{1 - x_0^2}}{\frac{\tan(\beta - \mu)}{\tan\alpha} + x_0} \qquad \text{- (20)}$$

In (20), $\in$ is the angle of the bend and is assumed to be 5.7^0. For fixed M' and varying *M*, the effect β and χ are shown in Table-10, Table-11 and Table-12.

Table-10 $M' = 2,\ M = 1.5$

β	0	0.5	1.25	1.5
χ	5.49	7.88	9.67	4.67

Table-11 $M' = 2,\ M = 2$

β	0	0.5	1.25	1.5
χ	5.45	9.68	11.74	12.27

Table-12 $M' = 2,\ M = 4$

β	0	0.5	1.25	1.5
χ	2.81	11.20	16.06	4.64

VI. CONCLUSIONS

The results are important contribution for aeronautical engineers. This contribution will be helpful for obtaining results in different situations.

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