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A THEORY OF TIE-SET PATH AND TIE-SET GRAPH -A GRAPH STUDY ON ROUTING NETWORK SYSTEM

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ABSTRACT

Aiming at establishment of graph theoretical basis of network management system using loops in the network as basic management units, this paper presents tie-set graph theory and its useful properties, such as transformation of tie-set graph, meta-tie-set graph, measure of tie-set graph and simplest tie-set graph. It is shown that proposed theory gives a theoretical basis and effectivk means for loop based network management. Distributed. Algorithms for finding lightest tie-set path are presenterr! that can apply to fault link avoidance and load balancing in an information network.

I. INTRODUCTION

The scale of the modern information network is becoming conspicuously larger and more complex, and the amount of information handled throughout the information network is rapidly increasing. Once an abrupt fault occurred on such a communication link, the amount of information that vanished by means of the fault is immeasurable. Besides, even if the link does not cause a fault, it is hard to avoid the concentration of traffic as far as the system is operated with an inflexible routing algorithm widely used in these days. Various methods to prevent such difficulties have been proposed and applied in practice[x]**,** most of which have limited capabilities. Therefore, it is desired to establish theoretical method for network management that enables the fault link avoidance and the effective load balancing systematically.**As** an example of applications **of** tie-set graph theory, the distributed algorithm for finding appropriate loop chain or "tie-set path" is presented, which will be applied to reliable routing protocol.In the next chapter, the theory of tie-set graph meta-tie-set graph and the simplest tie-set graph are presented. The problem of finding an optimum set of loops for network management is also discussed.

II. TIE-SET GRAPH THEORY

2.1 TIE-SET VECTOR SPACE AND TIE-SET BASIS - Let G=(V, E) be a given biconnected undirected graph with a set of vertices V=(vl, v2, ..., vn} and a set of edges E={el, e2, ..., em}.

Suppose a loop on G consist of a edge set {el, e2, ..., e k } , then L={el, e2, ..., ek} i s called a tie-set on G. A tie-set vector L is defined as follows.

$$L=(\varepsilon_1\varepsilon_2\varepsilon_3 \ldots\ldots \varepsilon_m) \qquad \varepsilon_k = \begin{cases} 0 & (e_k \notin L) \\ 1 & (e_k \in L) \end{cases}$$

Furthermore, the binary operations between arbitrary two tie-set vectors are defined as follows:

L, OL, = (&;,E;, . . . , E:,) @ (E / , E;, . . .,&A) = (E; 0 E / , & ; 0 E;, . ..,E:, 0 &A), = (E; E;, E; E;, . . . , E:, &A),

L, L, = (E; ,&; , . . .)& ,: (& / , E l , . . . ,E A)

Tie-set vector L is a binary vector. And the set of all possible tie-set vectors {L} on G forms a linear vector space A={ I,,} with respect to binary operation 0, and the dimension of A is ,u=IEl-lq+l, the nullity of G. In linear vector space with pdimension, there exists a basis, a system of ,u linear independent vectors. As for basis of tie-set vector space A, there are three categories as statedbellow.

[1] **Tie-set Basis B:** A system of *p* linear independent tie-set vectors in A.

[2] **Elementary Tie-set Basis Be:** A system of p linear independent elementary (node-disjoint) tie-set vectors in A.

[3] **Fundamental Tie-set Basis B):** A system of p tie-set vectors, each of which contains only one edge of a co-tree of a tree Ton G, or the fundamental system of tie-sets withrespect to tree T.Suppose {B}, {Be} and {Bf} are sets of all possiblecorresponding basis's of A, then the relation,{Bf}C{B"}G{B},h olds

2.2 TIE-SET GRAPH

For a given graph G, a tie-set graph is defined as a graph where each vertex corresponds to a tie-set of G and each edge is defined by the adjacent relation between two tie-sets.

Definition 2.11 Tie-Set Graph A tie-set graph Cj=(l', ZZ) is a graph with a set of vertices _V={LI, b, ..., Lp} where Li's are y linear independent tie-set vectors on G, and a set of edges E={&, Lj)}, From above definition, tieset graph is an imaginary graph with p vertices, which corresponds to p independent tie-sets of G, and edges;which correspond to the adjacent relations between two tie-sets Li and Lj of G. Since _V corresponds to a basis of G, i.e. _V=B, three categories of tie-set graph are obtained, namely

(1) Tie-set Graph G=(B, I?),

(2) Elementary Tie-set Graph C=(B'E, ")

(3) Fundamental Tie-set Graph Q=(BTEI).

Clearly, {g}C{Ge}G{G}A. fundamental tie-set graph of G corresponds to a sub-graph of the homeomorphic dual graph of G, provided that G is planar. Hence, the concept of tie-set graph can be interpreted as a generalization of dual graph.

2.3 L-TRANSFORMATION AND META-TIE-SET GRAPH

Any tie-set graph G=(B, I?) generated from a given gaph G can be. transformed to another tie-set graph G*=(B,E_),where B is obtained from B by an L-Transformation of basis defined as follows.

[Definition 2.31 L-Transformation of Basis Let basis B of tie-set vector space A on a Bven graph G be denoted as B={L1, L, ..., L, ..., Lj, ..., Lp}. Replace a tie-set vector L, in B with a new tie-sct vector b'=L;OLj, then a new basis B'={L1, b, ..., , ..., Lj, ..., Lp} is obtained. The operation to obtain B' from B, denoted as diJ)(B)=B*i,s called L-Transformation of basis B on tie-set vector Li with respect to Lj.+ This transformation corresponds to an elementary basis transformation of B. For simplicity, a L-Transformation of B to B is denoted as -C(B)=B.

[Definition 2.41 L-Transformation of Tie-set Graph. A transformation of a $e-set graph G=(B, E) into another tie-set graph G'=(B*, E) is dtfined as L-Transformation of tie-set graph G, where B is obtained from B by L-Transformation of B on tie-set vector L. with respect to L,, and represented as dj'l(G)=G*. Efge set E_' is determined by the relations of vertices of G accordingly. 6

[Definition 2.51 Meta-Tie-Set Graph Let Q={Cj} be a set of all possible tie-set graph G's generated from a given graph G, and CP ={(G, s)} be a set of edges which is defined between any pair of G. and s=L(sia) , tie-set graph obtained from Gi by once L-Transformation. Then the meta-tie-set graph is defined as T=(S2, a). + On a meta-tie-set graph r with respect to a tie-set graph G, the following property is obtained.

[Lemma 2.11 A meta-tie-set graph r is connected. +Furthermore, a meta-tie-set graph r has following hierarchical structure.

[Lemma 2.21 L-transformation of fundamental tie-set graph generates an elementary tie-set graph, i.e. L-transformation of an elementary tie-set graph generates a tie-bet graph, i.e. *{ f . (~) }C{G+}* { *.cQw* t *G7*.

2.4 THE SIMPLEST TIE-SET GRAPH

There exist many possible tie-set graphs derived from a biconnected graph. From the viewpoint of practical application, it is necessary to find out the most suitable tie-set graph for network management. For this purpose, a measure of a tie-set graph is introduced as sated in the following definition.

Definition 2.61 Measure of a Tie-Set GraphLet ***p (L)*** be a scalar function of a tie-set vector ***4*** then measure of a tie-set graph G By a scalar function ***p (L)*** of a tie-set vector **L,** we may consider the number or weight of edges and/or vertices belonging to L. Value of measure Q(E) corresponds to a complexity of a tie-set graph *C*, if *p (L)* represents the number of edges of a tie-set *L*, wefinition 2.71. The Simplest Tie-Set Graph For a given graph G, the tie-set graph G that minimizes *Q(G)* among all possible tie-set graphs is defined as **the** simplest tie-set graph. +The simplest tie-set graph plays an important role in loop based network management system. Therefore, the problem of finding simplest tie-set graph **E,** which has the minimum measure, is an essential issue in network management.

III. PROBLEM OF FINDINGTHE LIGHTEST TIE-SET PATH

3.1 TIE-SET PATH AND THE LIGHTEST TIE-SET PATH -Tie-set graph is a biconnected graph possessing ordinary graph theoretical properties such as path, loop, tree and so on. Among them, a path of a tie-set graph plays an important roll in network management.

Definition 3.11 Tie-Set Path of weights on components, edges andfor vertices, containing Pr(s, t) is defined as the weight of a tie-set path and denoted as W(Pr(S, t)). + There exist three types of tie-set path weights, namely:

(1) Edge weigbt W,(P,(s,t)):

(2) Vertex weight W4Pr(st,)):The total sum of weights of edges belonging to PL(st,) . The total sum of weights of vertices belonging to P&, t).

(3) Mixed weight WdPf(st,)): The total sum of weights of edges and vertices belonging to Pt(s,t).

Definition 3.31 The Lightest Tie-Set Path Among all possible tie-set paths from s to t in a tie-set graph _C, the minimum weight path with respect to appropriate weight is called lightest tie-set path. Lightest tie-set path with respect to WE(Pl(s,f)) corresponds to the traditional shortest path, while, those for WdPr(s, t)) or W,Pr(s, t)) are rather new concepts. In many practical applications, it is important to find the lightest path in a tie-set graph.Let (L,, ...,LJ be the sequence of tie-sets which correspond to vertices on a tie-set path Pr(s,t) then the following lemmas are obtained.

[Lemma 2.31 The graph Gp=L,U ... UL, is a subgraph of G, which corresponds to tie-set path P,(s,t). GP is biconnected. +

[Lemma 2.41 If the vertex weight of the path W4Pt(st,)) is minimum among all possible tie-set path from s to t, where vertices s and t include vertices v and vt in the an elementary loop including vertices v, and vr in G.+

3.2 THE LIGHTEST TIE-SET PATH PROBLEM

The problem of finding lightest tie-set path with respect to original graph G respectively, the graph G f = L,O ... OL, is - Fig. 3.1: To The Shortest Path Problem - kn elementary-path between vertex s and t (s z t) on a tie-set graph C, Pr(s, f), is defined as tie-setpath.

[Definition 3.21 Weight of Tie-Set Path In a tie-set path Pl(s,t) on a tie-set graph G, the total sum - vertex weight can be easily transformed into the well-known- shortest path problem by an appropriate transformation of a tie-set graph. Therefore, the lightest tie-set path problem can be easily solved in polynomial time, as in the case of shortest path problem. The transformation of a lightest graph problem with vertex weight to a shortest path problem is described as follows:

[1] Replace each edge with two edges having in the opposite directions.

[2] Add a vertex SI, and add an edge from the vertex s' to the source vertex s.

[3] Assign the weight *w(ek)* to the weight of each edge, where *w(ek)* is the weight of the terminal vertex of *ek.*

[4] Solve the shortest path problem using s' as the source vertex, instead of s.

VI. DISTRIBUTED ALGORITHMS FOR FINDING THE LIGHTEST TIE-SET PATH

4.1 AN ALGORITHM FOR FINDING THE LIGHTEST TIE-SET-PATH ON A TIE-SET GRAPH

A distributed algorithm for finding lightest tie-set paths from every vertex to every another vertex in tie-set graph for each entry in the tbl Begin if vi.tabk.measurek > tbl.measurek + vi.weight then begin vi.table.measurek := tblmeasurek -+ vi.weight;v,.table.parentk := v,; send ***vi.table*** to adjacent nodes ***{Lk},*** except ***v,;*** end;G is stated as follows. Hereafter terms node and **link** are used for information network, which correspond to vertex and edge of underlying graph discussed above,respectively.Every node ***vi*** in the tie-set graph has a table to determine the lightest tie-set paths. The table consists of entries for all nodes in **E.** Each entry for node ***vk*** haSparent,' measure] pair, where parent represents the previous node on the lightest tie-set path fiom the source node ***vk*** to ***vi,*** and measure is the weight of the path from ***vk*** to ***vi.*** For each ***vi's*** table, [parent, measure] pairs values are initially set ***[vi,*** the weight on ***vi]*** for ***vi's*** entry, ***[vi,*** 001 for all theothers. At first, each node sends its own table to adjacent nodes just once, and for each node ***y,*** the procedure (Fig. 4.1) is executed when it receives a table, a tbl, from ***v,.*** As a result, the lightest tie-set path from **s** to t is obtained through tracing the parent entry for **s** from node t.

4.2 AN ALGORITHM FOR FINDING LIGHTEST TIE-SET PATH ON ORIGINAL GRAPH

From the viewpoint of practical application, lightest tie-set paths finding algorithm on a tie-set graph *C* should be carried out on its original graph. The algorithm on a tie-set graph G described above can be converted to the corresponding algorithm on the original graph by taking the following conditions into account. For a given tie-set graph *C*, whose nodes correspond to the tie-sets in an original graph G. Hence, it is necessary to maintain the identical table on every node belonging to a tie-set. Anode in G maybe belongs to several nodes in G, and the edges in *G* are treated as imaginary edges in each node in G. Consequently, it is necessary to perform the operation of communication on edges in G by processing on a node in G.

V. APPLICATIONS OF TIE-SET GRAPH AND TIE-SET PATH

5.1 Tie-Set Graph Theory And Tie-Set Path In An Information Network

Tie-set graph and tie-set path is imaginary concepts that are supported by the state information at the nodes in an actual network. To obtain a tie-set path, or a chain of loops, it is necessary to construct the elementary tie-set basis beforehand in the network. The basis can be obtained through constructing the spanning tree and executing a distributed algorithm for L-Transformation. As stated in Lemma 2.4, the loop including arbitrary two vertices can be obtained from the corresponding tie-set path. This loop can be decomposed into the two node-disjoint paths, which can be utilized for main path and spare path, for forward path and backward path, or to gain broader bandwidth. Thus, tie-set path method can be applied various practical problems in information network as stated in the following sections.

5.2 Routing Protocol With Fault Links Avoiding The one **of** effective applications of tie-set path is the robust routing protocol that endures to keep connections even if failures occurred in the network. By use of the tie-set path, it is possible to construct more flexible fault tolerance network system than using two node-disjoint Detouring route around fault links-paths, since it can utilize many available links by avoiding fault links locally in each loop.

5.3 Load Balancing

Since tie-set path handles two node-disjoint paths and can split traffic load, it can prevent traffic over-concentration. Moreover, it is also possible to balance traffic load effectively in the network by making a detour to avoid congestion links in each loop. The distributed traffic management can be realized by applying the theory of tie-set graph and tie-set path to the information network.

VI. CONCLUSION

In order to establish the graph theoretical basis of network management such as fault link avoidance and load balancing, this paper proposed "tie-set graph theory". A tie-set graph is the graph whose vertices correspond to ***p*** (nullity of a given graph) independent tie-sets of a given graph. Various useful properties related tie-set graph, such as transformation of tie-set graph, meta-tie-set graph,measure of tie-set graph and simplest tie-set graph, are presented. The simplest tie-set graph is optimal for a management objective, where the set of tie-sets on the tie-set graph has the minimum measure of complexity. Theory of tie-set graph gives a theoretical basis and Effective means to realize the network management Utilizing loops as a basic management unit. An elementary path on tie-set graph, "tie-set path", is especially important, which corresponds to the chain of loops of the original graph. Tie-set path theory is applicable for a reliable routing and effective load balancing algorithms. Based on the proposed theory, outline of distributed algorithms for realizing fault link avoidance and load balancing in actual information network is presented. Further study on application of the theory to practical network management problems and development of distributed algorithms are left to the future.

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