INTUITIONISTIC FUZZY STRONGLY g*-CLOSED SETS

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ABSTRACT

In 1970, Levine introduced the concept of generalized closed sets in general topology. He observed that the family of all closed sets in a topological space X is a subfamily of the family of all generalized closed sets. He generalized some of well-known results of general topology replacing closed set by generalized closed sets, for instance, generalized closed subset of a compact space is compact and generalized closed subspace of a normal space is normal. Many authors utilized g-closed sets for the generalization of various topological concepts in general topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of fuzzy sets .In 1997 Coker introduced the concept of intuitionistic fuzzy topological spaces. Thakur and Chaturvedi introduced the notion of intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. After that different mathematicians worked and studied in different forms of intuitionistic fuzzy g-closed set and related topological properties. The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy strongly g- closed sets in intuitionistic fuzzy topological space. The class of all intuitionistic fuzzy strongly g*--closed sets lies between the class of all intuitionistic fuzzy w-closed sets and class of all intuitionistic fuzzy gpr-closed sets. We also introduce the concepts of intuitionistic fuzzy strongly g*-- open sets in intuitionistic fuzzy topological spaces.*

Key words: ***Intuitionistic fuzzy g-closed sets, Intuitionistic fuzzy g*-closed sets, Intuitionistic fuzzy strongly g*-closed sets and intuitionistic fuzzy strongly g*-open sets.***

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I INTRODUCTION

After the introduction of fuzzy sets by Zadeh [1] in 1965 and fuzzy topology by Chang [2] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [3] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [4] introduced the concept of intuitionistic fuzzy topological spaces. In 2000 Thakur and Chtuvedi introduced the concepts of intuitionistic fuzzy generalized closed sets[5] in intuitionistic fuzzy topology. After that many weak and strong forms of intuitionistic fuzzy g-closed sets such as intuitionistic fuzzy rg-closed sets[6],intuitionistic fuzzy sg-closed sets[7], intuitionistic fuzzy g*-closed sets[8], intuitionistic fuzzy αg-closed sets[9], intuitionistic fuzzy gα-closed sets[10], intuitionistic fuzzy w-closed sets[11],intuitionistic fuzzy rw-closed sets[12], intuitionistic fuzzy gpr-closed sets[13] intuitionistic fuzzy rgα-closed sets[14] intuitionistic fuzzy spg-closed sets [15], intuitionistic fuzzy gsp-[16]closed sets and intuitionistic fuzzy gp[17] have been appeared in the literature.

In the present paper we extend the concepts of strongly g*-closed sets due to T.Rajendrakumar and G. Anandajothi [18] in intuitionistic fuzzy topological spaces . The class of intuitionistic fuzzy strongly g*-closed sets is properly placed between the class of intuitionistic fuzzy w-closed sets and intuitionistic fuzzy gpr-closed sets. We also introduced the concepts of intuitionistic fuzzy strongly g*-open sets, and obtain some of their characterization and properties.

II PRELIMINARIES

Let X be a nonempty fixed set. An intuitionistic fuzzy set A[3] in X is an object having the form $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\Upsilon_A : X \to [0,1]$ denotes the degree of membership $\mu_A(x)$ and the degree of non membership $\gamma_A(x)$ of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$. The intutionistic fuzzy sets $\tilde{\mathbf{0}} = \{< x, 0, 1 > : x \in X\}$ and $\tilde{\mathbf{1}} = \{<x, 1, 0> : x \in X\}$ are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is called a subset of an intuitionistic fuzzy set $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ (for short $A \subseteq B$) if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for each $x \in X$. The complement of an intuitionistic fuzzy set $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ is the intuitionistic fuzzy set $A^c = \{<x, \gamma_A(x), \mu_A(x)> : x \in X\}$. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets $A_i = \{< x, \mu_{Ai}(x), \gamma_{Ai}(x) > : x \in X, (i \in \wedge)\}$ of X be the intuitionistic fuzzy set $\cap A_i = \{<x, \wedge \mu_{Ai}(x), \vee \gamma_{Ai}(x) > : x \in X\}$ (resp. $\cup A_i = \{<x, \vee \mu_{Ai}(x), \wedge \gamma_{Ai}(x) > : x \in X\}$). Two intuitionistic fuzzy sets $A = \{<x, \mu_A(x), \gamma_A(x)> : x \in X\}$ and $B = \{<x, \mu_B(x), \gamma_B(x)> : x \in X\}$ are said be q-coincident (A_qB for short) if and only if $\exists$ an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$. A family $\Im$ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology [6]on X if the intuitionistic fuzzy sets $\tilde{\mathbf{0}}, \tilde{\mathbf{1}} \in \Im$, and $\Im$ is closed under arbitrary union and finite intersection. The ordered pair $(X,\Im)$ is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in $\Im$ is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set .The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted int(A) [4].

Lemma 2.1 [4]**:** Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space $(X, \Im)$. Then:

(a) $(A_q B) \Leftrightarrow A \subseteq B^c$.

(b) A is an intuitionistic fuzzy closed set in X $\Leftrightarrow$ cl (A) = A

(c) A is an intuitionistic fuzzy open set in X $\Leftrightarrow$ int (A) = A.

(d) $cl\,(A^c) = (int\,(A))^c$.

(e) $int\,(A^c) = (cl\,(A))^c$.

(f) $A \subseteq B \Rightarrow int\,(A) \subseteq int\,(B)$.

(g) $A \subseteq B \Rightarrow cl\,(A) \subseteq cl\,(B)$.

(h) $cl\,(A \cup B) = cl\,(A) \cup cl(B)$.

(i) $int(A \cap B) = int\,(A) \cap int(B)$

Definition 2.1 [19]**:** Let X is a nonempty set and $c \in X$ a fixed element in X. If $\alpha \in (0, 1]$ and $\beta \in [0, 1)$ are two real numbers such that $\alpha+\beta \leq 1$ then:

(a) $c(\alpha,\beta) = < x, c_\alpha, c_{1-\beta} >$ is called an intuitionistic fuzzy point in X, where α denotes the degree of membership of $c(\alpha,\beta)$, and β denotes the degree of non membership of $c(\alpha,\beta)$.

(b) $c(\beta) = < x, 0, 1-c_{1-\beta} >$ is called a vanishing intuitionistic fuzzy point in X, where β denotes the degree of non membership of $c(\beta)$.

Definition 2.2[4]: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called:

(a) An intuitionistic fuzzy semi open of X if there is an intuitionistic fuzzy set O such that $O \subseteq A \subseteq cl(O)$.

(b) An intuitionistic fuzzy semi closed if the compliment of A is an intuitionistic fuzzy semi open set.

(c) An intuitionistic fuzzy regular open of X if int(cl(A)) = A .

(d) An intuitionistic fuzzy regular closed of X if cl(int(A))= A.

(e) An intuitionistic fuzzy pre open if $A \subseteq int\,(cl(A))$.

(f) An intuitionistic fuzzy pre closed if $cl(int(A)) \subseteq A$

(g) An intuittionistic fuzzy α-open $A \subseteq int\,(cl(intA))$

(h) intuitionistic fuzzy α- closed if $cl(int(cl(A))) \subseteq A$

Definition 2.3[4] If A is an intuitionistic fuzzy set in intuitionistic fuzzy topological space$(X,\Im)$ then

(a) scl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy semi closed}
(b) pcl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy pre closed}
(c) αcl (A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy α closed}
(d) spcl(A) = $\cap$ { F: A $\subseteq$ F, F is intuitionistic fuzzy semi pre-closed}

Definition 2.4: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is called:

(a) Intuitionistic fuzzy g-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[5]
(b) Intuitionistic fuzzy rg-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[6]
(c) Intuitionistic fuzzy sg-closed if scl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[7]
(d) Intuitionistic fuzzy g*-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g-open.[8]
(e) Intuitionistic fuzzy αg-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy - open.[9]
(f) Intuitionistic fuzzy gα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy α- open.[10]
(g) Intuitionistic fuzzy w-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[11]
(h) Intuitionistic fuzzy rw-closed if cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular semi open.[12]
(i) Intuitionistic fuzzy gpr-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular open.[13]
(j) Intuitionistic fuzzy rgα-closed if αcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy regular α-open.[14]
(k) Intuitionistic fuzzy spg-closed if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy semi open.[15]
(l) Intuitionistic fuzzy gsp-closed if spcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy - open.[16]
(m) Intuitionistic fuzzy gp-closed if pcl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[17]
(n) Intuitionistic fuzzy gs-closed if scl (A) $\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy open.[20]
The complements of the above mentioned closed set are their respective open sets.

Remark 2.2:

(a) Every intuitionistic fuzzy closed set is intuitionistic fuzzy g-closed set.[5]
(b) Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy αg-closed set.[9]
(c) Every intuitionistic fuzzy g-closed is intuitionistic fuzzy gα-closed set.[10]
(d) Every intuitionistic fuzzy αg-closed is intuitionistic fuzzy gα-closed set.[10]
(e) Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy g-closed [11]
(f) Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy sg-closed set.[11]
(g) Every intuitionistic fuzzy regular closed is intuitionistic fuzzy rw-closed set.[12]
(h) Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy rw-closed set.[12]

(i) Every intuitionistic fuzzy rg-closed set is intuitionistic fuzzy gpr-closed. [13]
(j) Every intuitionistic fuzzy αg-closed is intuitionistic fuzzy gpr-closed. [13]
(k) Every intuitionistic fuzzy sg-closed set is intuitionistic fuzzy spg-closed set.[15]
(l) Every intuitionistic fuzzy spg-closed set is intuitionistic fuzzy gsp-closed set.[16]
(m) Every intuitionistic fuzzy pre-closed set is intuitionistic fuzzy gp-closed set [17]
(n) Every intuitionistic fuzzy gp-closed set is intuitionistic fuzzy gsp-closed set.[17]
(o) Every intuitionistic fuzzy sg-closed set is intuitionistic fuzzy gs-closed set.[20]

III INTUITIONISTIC FUZZY STRONGLY g* -CLOSED SET

Definition 3.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space $(X,\Im)$ is called an intuitionistic fuzzy strongly g*-closed if cl (int(A))$\subseteq$ O whenever A $\subseteq$ O and O is intuitionistic fuzzy g-open in X.

First we prove that the class of intuitionistic fuzzy strongly g*- closed sets properly lies between the class of intuitionistic fuzzy w-closed sets and the class of intuitionistic fuzzy gpr-closed sets.

Theorem 3.1: Every intuitionistic fuzzy w-closed set is intuitionistic fuzzy strongly g*-closed -closed.

Proof: Let A is intuitionistic fuzzy w-closed set. Let A $\subseteq$ U and U intuitionistic fuzzy semi-open sets in X. Since every intuitionistic fuzzy semi open set is intuitionistic fuzzy g-open sets U is intutionistic fuzzy g-open sets. Now by definition of intuitionistic fuzzy w-closed sets cl(A) $\subseteq$ U. But cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U. We have cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy strongly g*-closed set.

Remark 3.1: The converse of above theorem need not be true as from the following example.

Example 3.1: Let X = {a, b} and $\Im = \{\tilde{\mathbf{0}}, U, \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X, where U = {< a,0.7,0.2>,< b, 0.6, 0.3> }.Then the intuitionistic fuzzy set A = {<a,0.7,0.2>,<b,0.8,0.1>} is intuitionistic fuzzy strongly g* -closed but it is not intuitionistic fuzzy w- closed.

Theorem 3.2: Every intuitionistic fuzzy rw-closed set is intuitionistic fuzzy strongly g*-closed -closed.

Proof: Let A is intuitionistic fuzzy rw-closed set. Let A $\subseteq$ U and U intuitionistic fuzzy regular semi-open sets in X. Since every intuitionistic fuzzy regular semi open set is intuitionistic fuzzy g-open sets U is intutionistic fuzzy g-open sets. Now by definition of intuitionistic fuzzy rw-closed sets cl(A) $\subseteq$ U. But cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U. We have cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy strongly g*-closed set.

Remark 3.2: The converse of above theorem need not be true as from the following example.

Example 3.2: Let X = {a, b} and $\Im$ = {$\tilde{\mathbf{0}}$,U $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X, where U = {< a,0.5,0.3>,< b, 0.4, 0.3> }.Then the intuitionistic fuzzy set A = {<a,0.5,0.2>,<b,0.7,0.1>} is intuitionistic fuzzy strongly g* -closed but it is not intuitionistic fuzzy rw- closed.

Theorem 3.3: Every intuitionistic fuzzy g*-closed set is intuitionistic fuzzy strongly g*--closed sets..

Proof: Let A is intuitionistic fuzzy g*-closed set. Let .A $\subseteq$ U and U is intuitionistic fuzzy g –open sets in X. Now by definition of intuitionistic fuzzy g*-closed sets cl(A) $\subseteq$ U. But cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U. We have cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy strongly g*-closed set.

Remark 3.3: The converse of above theorem need not be true as from the following example.

Example 3.3: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U, V,W defined as follows

O = { < a,0.9,0.i> ,<b,0,1>,<c,0,1>,<d,0,1>}

U = { < a,0,1,> ,<b,0.8,0.1>,<c,0,1>,<d,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1,>,<c,0,1>,<d,0,1>}

W = { < a,0.9,0.1,> ,<b,0.8,0.1>,<c,0.7,0.2>,<d,0,1>}

$\Im$ = {$\tilde{\mathbf{0}}$, O, U, V, W, $\tilde{\mathbf{1}}$} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a,0,1>,<b,0,1> <c,0.7,0.2>,<d,0,1>} is intuitionistic fuzzy strongly g* -closed but it is not intuitionistic fuzzy g*-closed.

Theorem 3.4: Every intuitionistic fuzzy g-closed set is intuitionistic fuzzy strongly g*--closed sets.

Proof: Let A is intuitionistic fuzzy g-closed set. Let A $\subseteq$ U and U intuitionistic fuzzy -open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g-open sets U is intutionistic fuzzy g-open sets. Now by definition of intuitionistic fuzzy g-closed sets cl(A) $\subseteq$ U. But cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U. We have cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy strongly g*-closed set.

Remark 3.4: The converse of above theorem need not be true as from the following example

Example 3.4: Let X = {a, b, c, d, e} and intuitionistic fuzzy sets O,U, V defined as follows

O = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0,1>,<d,0,1> <e,0,1>}

U = { < a,0,1,> ,<b,0,1>,<c,0.8,0.1>,<d,0.7,0.2> <e,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1>,<c,0.8,0.1>, <d,0.7,0.2>,<e,0,1>}

Let $\Im$ = {$\tilde{\mathbf{0}}$, O, U, V, $\tilde{\mathbf{1}}$ } be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A ={<a,0.9,0.1 >,<b,0,1 >,<c,0,1>,<d,0,1>,<e,0,1>} is intuitionistic fuzzy strongly g* -closed but it is not intuitionistic fuzzy g-closed.

Theorem 3.5: Every intuitionistic fuzzy αg-closed set is intuitionistic fuzzy strongly g*--closed sets.

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Proof: Let A is intuitionistic fuzzy αg-closed set. Let $A \subseteq U$ and U intuitionistic fuzzy -open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g-open sets U is intutionistic fuzzy g-open sets. Now by definition of intuitionistic fuzzy αg-closed sets $\alpha cl(A) \subseteq U$. But $\alpha cl(A) \subseteq cl(A)$ therefore $cl(A) \subseteq A$. Now $cl(int(A)) \subseteq cl(A) \subseteq U$. We have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy strongly g*-closed set.

Remark 3.5: The converse of above theorem need not be true as from the following example

Example 3.5: Let X = {a, b, c, d} and intuitionistic fuzzy sets O,U defined as follows

O = { < a,0.7,0.1> ,<b,0.6,0.2>,<c,0,1>,<d,0,1> }

U = { < a,0,1,> ,<b,0,1>,<c,0.7,0.2>,<d,0.6,0.1> }

Let $\Im = \{\tilde{\mathbf{0}}, O, U, \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A ={<a,0.7,0.1 >,<b,0,1 >,<c,0,1>,<d,0,1>} is intuitionistic fuzzy strongly g* -closed but it is not intuitionistic fuzzy αg-closed.

Theorem 3.6: Every intuitionistic fuzzy gα-closed set is intuitionistic fuzzy strongly g*--closed sets.

Proof: It follows from theorem 3.5 the fact that every intuitionistic fuzzy gα-closed set is intuitionistic fuzzy αg-closed sets.

Theorem 3.7: Every intuitionistic fuzzy gsp-closed set is intuitionistic fuzzy strongly g*--closed sets.

Proof: Let A is intuitionistic fuzzy gsp-closed set. Let $A \subseteq U$ and U intuitionistic fuzzy -open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g-open sets U is intutionistic fuzzy g-open sets. Now by definition of intuitionistic fuzzy gsp-closed sets $spcl(A) \subseteq U$. But $spcl(A) \subseteq cl(A)$ therefore $cl(A) \subseteq A$. Now $cl(int(A)) \subseteq cl(A) \subseteq U$. We have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy strongly g*-closed set.

Remark 3.6: The converse of above theorem need not be true as from the following example.

Example 3.6: Let X = {a, b} and $\Im = \{\tilde{\mathbf{0}}, U\ \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X, where U={<a,0.5,0.3>,<b,0.2,0.3>}.Then the intuitionistic fuzzy set A = {<a,0.2,0.4>,<b,0.6, 0.1>} is intuitionistic fuzzy strongly g* -closed but it is not intuitionistic fuzzy gsp- closed.

Theorem 3.8: Every intuitionistic fuzzy spg-closed set is intuitionistic fuzzy strongly g*--closed sets.

Proof: It follows from theorem 3.7 the fact that every intuitionistic fuzzy spg-closed set is intuitionistic fuzzy gsp-closed sets.

Theorem 3.9: Every intuitionistic fuzzy gp-closed set is intuitionistic fuzzy strongly g*--closed sets.

Proof: Let A is intuitionistic fuzzy gp-closed set. Let $A \subseteq U$ and U intuitionistic fuzzy -open sets in X. Since every intuitionistic fuzzy open set is intuitionistic fuzzy g-open sets U is intutionistic fuzzy g-open sets. Now by definition of intuitionistic fuzzy gp-closed sets pcl(A) $\subseteq$ U. But pcl(A)$\subseteq$ cl(A) therefore cl(A)$\subseteq$ A. Now cl(int(A)) $\subseteq$ cl(A) $\subseteq$ U. We have cl(int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is intuitionistic fuzzy g-open in X. Therefore A is intuitionistic fuzzy strongly g*-closed set.

Remark 3.7: The converse of above theorem need not be true as from the following example.

Example 3.7: Let X = {a, b} and $\Im = \{\tilde{\mathbf{0}}, U\ \tilde{\mathbf{1}}\}$ be an intuitionistic fuzzy topology on X, where U={<a,0.4,0.3>,<b,0.6,0.3>}.Then the intuitionistic fuzzy set A = {<a,0.4,0.2>,<b,0.6, 0.1>} is intuitionistic fuzzy strongly g* -closed but it is not intuitionistic fuzzy gp- closed.

Theorem 3.10: If intuitionistic fuzzy set A is both intuitionistic fuzzy open and Intuitionistic fuzzy strongly g*-closed sets then A is intuitionistic fuzzy rg-open sets.

Proof: Let intuitionistic fuzzy A is both open and intuitionistic fuzzy strongly g*-closed sets. Let A$\subseteq$ U where U is regular open set. Since every intuitionistic fuzzy regular open sets is intuitionistic open and therefore intuitionistic fuzzy g-open sets. Then by definition of intuitionistic fuzzy strongly g*-closed sets, we have cl(int(A))$\subseteq$ A. Now A is intuitionistic fuzzy open sets which implies that int(A) = A. Hence cl(A)$\subseteq$ A whenever A$\subseteq$ U where U is regular open set. Hence A is intuitionistic fuzzy rg-closed set.

Theorem 3.11: If intuitionistic fuzzy set A is both intuitionistic fuzzy open and Intuitionistic fuzzy strongly g*-closed sets then A is intuitionistic fuzzy gpr-open sets.

Proof: Let intuitionistic fuzzy A is both open and intuitionistic fuzzy strongly g*-closed sets. Let A$\subseteq$ U where U is regular open set. Since every intuitionistic fuzzy regular open sets is intuitionistic open and therefore intuitionistic fuzzy g-open sets. Then by definition of intuitionistic fuzzy strongly g*-closed sets, we have cl(int(A))$\subseteq$ A. Now A is intuitionistic fuzzy open sets which implies that int(A) = A. Therefore cl(A)$\subseteq$ A But pcl(A) $\subseteq$ Cl(A). Which implies that pcl(A) $\subseteq$ Cl(A))$\subseteq$ A. We have pcl(A) $\subseteq$ A whenever A$\subseteq$ U where U is regular open set. Hence A is intuitionistic fuzzy gpr-closed set.

Corollary 3.1: Every intuitionistic fuzzy closed set is intuitionistic fuzzy strongly g*-closed set.

Proof: It follows from remark 2.2(a) and theorem 3.4.

Corollary 3.2: Every intuitionistic fuzzy α-closed set is intuitionistic fuzzy strongly g*-closed set.

Proof: It follows from remark 2.2(b) and theorem 3.5.

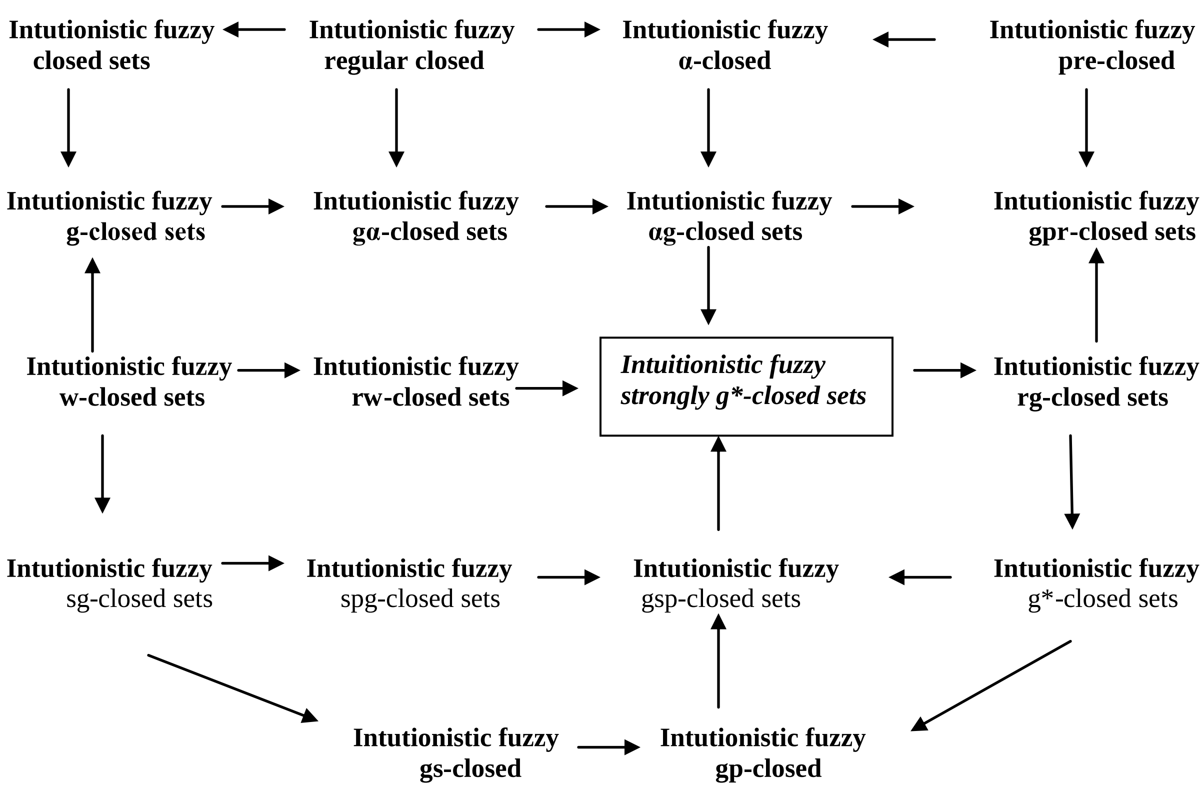
Corollary 3.3: Every intuitionistic fuzzy pre-closed set is intuitionistic fuzzy strongly g*-closed set.

Proof: It follows from remark 2.2(m) and theorem 3.9.

Corollary3.4: Every intuitionistic fuzzy regular-closed set is intuitionistic fuzzy strongly g*-closed set.

Proof: It follows from remark 2.2(g) and theorem 3.2.

Remark 3.8: From the above discussion and known results we have the following diagram of implications:



Theorem 3.12: Let $(X,\Im)$ be an intuitionistic fuzzy topological space and A is an intuitionistic fuzzy set of X. Then A is intuitionistic fuzzy strongly g*-closed if and only if $\rceil(AqF) \Rightarrow \rceil(cl(int((A))qF)$ for every intuitionistic fuzzy g- closed set F of X.

Proof: **Necessity**: Let F be an intuitionistic fuzzy g- closed set of X and $\rceil(AqF)$. Then by Lemma 2.1(a), $A \subseteq F^c$ and F^c intuitionistic fuzzy g- open in X. Therefore $cl(int(A)) \subseteq F^c$ by Def 3.1 because A is intuitionistic fuzzy strongly g*-closed. Hence by lemma 2.1(a), $\rceil(cl\,(int(A))qF)$.

Sufficiency: Let O be an intuitionistic fuzzy g- open set of X such that $A \subseteq O$ i.e. $A \subseteq ((O)^c)^c$ Then by Lemma 2.1(a), $\rceil(A_q O^c)$ and O^c is an intuitionistic fuzzy g- closed set in X. Hence by hypothesis $\rceil(cl\,(int(A))_q O^c)$. Therefore by Lemma 2.1(a), cl (int(A))$\subseteq$((O)^c)^c i .e. cl (int(A)) $\subseteq$ O Hence A is intuitionistic fuzzy strongly g*-closed in X.

Remark 3.9: The intersection of two intuitionistic fuzzy strongly g*-closed sets in an intuitionistic fuzzy topological space (X,$\Im$) may not be intuitionistic fuzzy strongly g*-closed. For,

Example 3.8: Let X = {a, b, c, d } and intuitionistic fuzzy sets O,U, V,W defined as follows

O = { < a,0.9,0.1> ,<b,0,1>,<c,0,1>,<d,0,1>}

U = { < a,0,1,> ,<b,0.8,0.1>,<c,0,1>,<d,0,1>}

V = { < a,0.9,0.1> ,<b,0.8,0.1,>,<c,0,1>,<d,0,1>}

W = { < a,0.9,0.1,> ,<b,0.8,0.1>,<c,0.7,0.2>,<d,0,1>}

$\Im$ = {$\tilde{0}$, O, U, V, W $\tilde{1}$,} be an intuitionistic fuzzy topology on X. Then the intuitionistic fuzzy set A = {<a,0.9,0.1>,<b,0.8,0.1> <c,0,1>,<d,0,1>} and

B = {<a, 0.9, 0.1 >, < b, 0, 1>, < c, 0.7, 0.2> ,<d,0.9,0.1>}

are intuitionistic fuzzy strongly g*-closed in (X,$\Im$) but A $\cap$ B is not intuitionistic fuzzy strongly g*-closed.

Theorem 3.13: Let A be an intuitionistic fuzzy strongly g*-closed set in an intuitionistic fuzzy topological space (X,$\Im$) and A $\subseteq$ B $\subseteq$ cl (int(A)). Then B is intuitionistic fuzzy strongly g*-closed in X.

Proof: Let O be an intuitionistic fuzzy g-open set in X such that B $\subseteq$ O. Then A $\subseteq$ O and since A is intuitionistic fuzzy strongly g*-closed, cl(int(A)) $\subseteq$ O. Now B $\subseteq$ cl (int(A)) $\Rightarrow$ cl(int(B)) $\subseteq$ cl(int(cl(Int(A)))) = cl(int(A)), cl (int(B)) $\subseteq$ cl (int(A)) $\subseteq$ O. Consequently B is intuitionistic fuzzy strongly g*-closed**.**

Definition 3.2: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is called intuitionistic fuzzy g*-open if and only if its complement A^c is intuitionistic fuzzy strongly g*-closed.

Remark 3.10: Every intuitionistic fuzzy w-open set is intuitionistic fuzzy strongly g*-open but its converse may not be true.

Example 3.9: Let X = {a, b} and $\Im$ = {$\tilde{\mathbf{0}}$, U,$\tilde{\mathbf{1}}$, } be an intuitionistic fuzzy topology on X, where U={<a,0.7,0.2>,<b,0.6,0.3>}.Then the intuitionistic fuzzy set A={<a,0.2,0.7>,<b,0.1,0.8>} is intuitionistic fuzzy strongly g*-open in (X, $\Im$) but it is not intuitionistic fuzzy w-open in (X, $\Im$).

Theorem 3.14: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,$\Im$) is intuitionistic fuzzy strongly g*-open if F $\subseteq$ cl(int (A)) whenever F is intuitionistic fuzzy g-closed and F $\subseteq$ A.

Proof: Follows from definition 3.1 and Lemma 2.1

Theorem 3.15: Let A be an intuitionistic fuzzy strongly g*-open set of an intuitionistic fuzzy topological space $(X,\mathfrak{I})$ and $cl(int(A)) \subseteq B \subseteq A$. Then B is intuitionistic fuzzy strongly g*-open.

Proof: Suppose A is an intuitionistic fuzzy strongly g*-open in X and $cl(int(A)) \subseteq B \subseteq A$. $\Rightarrow A^c \subseteq B^c \subseteq (cl(int(A)))^c$ $\Rightarrow A^c \subseteq B^c \subseteq cl(int(A^c))$ by Lemma 2.1(d) and A^c is intuitionistic fuzzy strongly g*-closed it follows from theorem that B^c is intuitionistic fuzzy strongly g*-closed . Hence B is intuitionistic fuzzy strongly g*-open.

V CONCLUSION

The theory of g-closed sets plays an important role in general topology. Since its inception many weak and strong forms of g-closed sets have been introduced in general topology as well as fuzzy topology and intuitionistic fuzzy topology. The present paper investigated a new weak form of intuitionistic fuzzy g-closed sets called intuitionistic fuzzy strongly g*-closed sets which contain the classes of intuitionistic fuzzy closed sets, intuitionistic fuzzy pre closed sets, intuitionistic fuzzy α-closed sets, intuitionistic fuzzy w-closed sets, intuitionistic fuzzy rw-closed sets, intuitionistic fuzzy gp-closed sets , intuitionistic fuzzy gpr-closed sets , intuitionistic fuzzy αg-closed sets, intuitionistic fuzzy gα-closed sets , intuitionistic fuzzy spg-closed sets, intuitionistic fuzzy gsp-closed sets , intuitionistic fuzzy g*- closed sets and contained in the classes of fuzzy rg-closed sets and fuzzy gpr-closets. The class of all intuitionistic fuzzy strongly g*--closed sets lies between the class of all intuitionistic fuzzy w-closed sets and class of all intuitionistic fuzzy gpr-closed sets. Several properties and application of intuitionistic fuzzy strongly g*--closed sets are studied. Many examples are given to justify the result.

REFERENCES

1. L.H. Zadeh , Fuzzy Sets, *Information and Control, 18,* 1965, 338-353 **.**
2. C.L.Chang Fuzzy Topological Spaces *J. Math. Anal. Appl. 24* , 1968, 182-190.
3. K. Atanassova and S. Stoeva Intuitionistic Fuzzy Sets, *In Polish Symposium on Interval and Fuzzy Mathematics* , *Poznan,* , 1983, 23-26.
4. Coker D. An Introduction to Inuitionistic Fuzzy Topological Spaces , *Fuzzy Sets and Systems 88* ,1997, 81-89
5. S.SThakur . and Rekha Chaturvedi , Generalized closed set in intuitionistic fuzzy topology, *The journal of Fuzzy Mathematics 16(3)* , 2008, 559-572.
6. S.S. Thakur and Rekha Chaturvedi , Regular generalized closed sets in intuitionistic fuzzy topology, *Studii Si Cercetari Stiintifice Seria Mathematica.16* 2010, ,257-272.
7. S.S. Thakur and Jyoti Pandey Bajpai, Semi generalized closed sets in intuitionistic fuzzy topology, *International Review of Fuzzy Mathematics, 6(2), 69-76,* 2011.
8. Rekha Chaturvedi , Some classes of generalized closed sets in intuitionistic fuzzy topological spaces , Doctoral Dissertation , Rani Durgavati Vishwavidyalaya Jabalpur , M.P. , India, 2008.

9. K. Sakthivel , Intuitionistic fuzzy Alpha generalized continuous mappings and Intuitionistic fuzzy Alpha generalized irresolute mappings, *Applied Mathematical Sciences, 4,* 2010, 1831-1842.
10. D.. Kalamani , Sakthinel and C,S. Gowri . , Generalized Alpha closed set in intuitionistic fuzzy topological spaces, *Applied Mathematical Sciences, 6(94),* 2012, 4692- 4700.
11. S.S. Thakur and Jyoti Pandey Bajpai Intuitionistic fuzzy w-closed sets and intuitionistic fuzzy w-continuity, *International Journal of Contemporary Advanced Mathematics*, 1(1), 2010, 1-15.
12. S.S. Thakur and Jyoti Pandey Bajpai Intuitionistic fuzzy rw-closed sets and intuitionistic fuzzy rw-continuity, *Notes on Intuitionistic Fuzzy Sets, 17(2),* 2011 82-96.
13. S.S. Thakur and Jyoti Pandey Bajpai On Intuitionistic fuzzy gpr-closed sets , *Fuzzy Information and Engineering, Springer , 4,* 2012, 425-444.
14. S.S. Thakur and Jyoti Pandey Bajpai Intuitionistic fuzzy rgα-closed sets , *International Journal of Fuzzy Systems and Rough Systems, 4(1),* 2011, 59-65.
15. M. Thirumalaiswamy and K. Ramesh, Semi pre generalized closed sets in intuitionistic fuzzy topological spaces, *Inter. J. Math. Archive. 4(2),* 2013, 1-5.
16. R. Santhi and D.Jyanthi Intuitionistic fuzzy generalized semi pre closed sets, *Tripura Math. Soci.* 2009, 61-72.
17. P. Rajarajeswari and L. Senthil Kumar, Generalized pre-closed sets in intuitionistic fuzzy topological spaces, *International journal of Fuzzy Mathematics and Systems, 3,* 2011, 253–262.
18. *T.Rajendrakumar1* and *G.Anandajothi2* On Fuzzy Strongly g*-Closed Sets in Fuzzy Topological Spaces *Intern. J. Fuzzy Mathematical Archive . 3,* 2013, *68-75*
19. D, Coker and M. Demirci, On Intuitionistic Fuzzy Points, *Notes On IFS :2(1),* 1995, 78-83.
20. R. Santhi, . and K. Sakthivel, Intuitionistic fuzzy generalized semi continuous mappings, *Advances in Theoretical and Applied Mathematics, 5* ,2009 73-82