VERTEX EQUITABLE LABELING OF UNION OF CYCLIC SNAKE IN TRANSFORMED TREES

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ABSTRACT

Let G be a graph with p vertices and q edges and A = $\left\{0,1,2,\ldots,\left\lceil \frac{q}{2} \right\rceil\right\}$. *A vertex labeling f: V (G)* $\to$ *A induces an edge labeling* f^* *defined by* $f^*(uv) = f(u) + f(v)$ *for all edges uv. For* $a \in A$, *let* $v_f(a)$ *be the number of vertices v with f(v) = a. A graph G is said to be vertex equitable if there exists a vertex labeling f such that for all a and b in A,* $\left|v_f(a) - v_f(b)\right| \le 1$ *and the induced edge labels are 1, 2, 3,…,q. In this paper, we prove that* $T\,\hat{O}\,nC_4$ and zig - zag triangle *are vertex equitable graphs.*

Key Words: Vertex Equitable Labeling, Vertex Equitable Graph

AMS Classification (2010): 05C78

I INTRODUCTION

All graphs considered here are simple, finite, connected and undirected .We follow the basic notations and terminologies of graph theory as in [1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey of graph labeling can be found in [2]. The vertex set and the edge set of a graph are denoted by *V*(*G*) and *E*(*G*) respectively. The concept of vertex equitable labeling was due to Lourdusamy and Seenivasan in [3] and further studied in [4-12]. Let *G* be a graph with *p* vertices and *q* edges and *A*= $\left\{0,1,2,\ldots,\left\lceil \frac{q}{2} \right\rceil\right\}$. A graph *G* is said to be vertex equitable if there exists a vertex labeling $f : V(G) \to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges *uv* such that for all *a* and *b* in *A*, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are 1, 2, 3,…, *q*, where $v_f(a)$ be the number of vertices *v* with $f(v) = a$ for $a \in A$. The vertex labeling *f* is known as vertex equitable labeling. A graph *G* is said to be a vertex equitable if it admits vertex equitable labeling. In this paper we extend our study on vertex equitable labeling and prove that $T\,\hat{O}\,nC_4$ and zig - zag triangle are vertex equitable graphs.

We use the following definitions in the subsequent section.

Definition 1.2. Let $G_1, G_2, \ldots, G_n, n \geq 2$ be *n* graphs and u_i be a vertex of G_i for $1 \leq i \leq n$. The graph obtained by adding an edge between u_i and u_{i+1} for $1 \leq i \leq n-1$ is called a path union of $G_1, G_2, \ldots, G_n$ and is denoted by $P(G_1, G_2, \ldots, G_n)$. When all the *n* graphs are isomorphic to a graph G, it is denoted by $P(n.G)$.

Definition 1.3. [13] A nC_k-snake is defined as a connected graph in which all the *n*-blocks are isomorphic to the cycle C_k and the block-cut point graph is a path. Let P be the path of minimum length that contains all the cut vertices of a nC_k –snake. Any nC_k-snake can be represented by a string $s_1, s_2, \ldots, s_{n-2}$ of integers of length *n*-2 where the i^{th} integer, s_i, on the string is the distance between i^{th} and $(i+1)^{th}$ cut vertices on the path *P* from one extreme and is taken from $S_k=\{1,2,\ldots,\left\lfloor \frac{k}{2} \right\rfloor\}$. The strings obtained for both extremes are assumed to be the same. For example, the string of a $10C_4$-snake is shown in Figure 1.1 is 2,2,1,2,1,1,2,1.

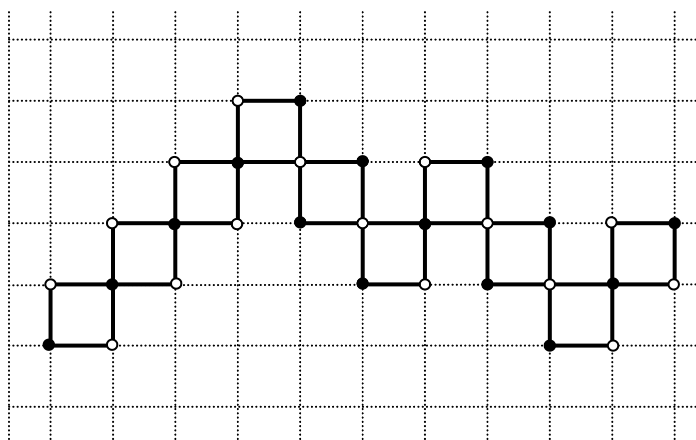

Figure 1.1: An Embedding of $10C_4$ –Snake

A nC_k –snake is said to be linear if each integer of its string is $\left\lfloor \frac{k}{2} \right\rfloor$.The nC_4 –snake graph with diagonal vertices are u_{1j} $(1 \leq j \leq n+1)$, left to the diagonal vertices are v_{1j} $(1 \leq j \leq n)$ and right to the diagonal vertices are w_{1j} $(1 \leq j \leq n)$.

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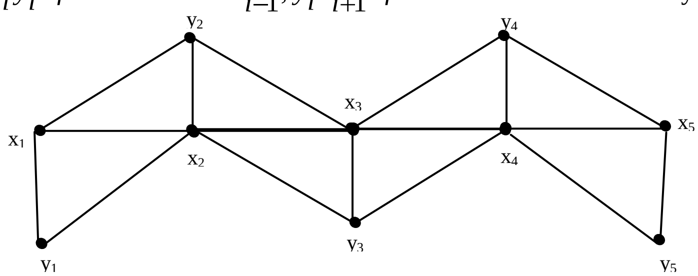
Definition 1.4: Let *H* be any graph with *n* vertices $v_1, v_2, \ldots, v_n$ and let $G_1, G_2, \ldots, G_n$ be n graphs. Then $H\,\hat{O}\,(G_1, G_2, \ldots, G_n)$ is a graph obtained by identifying a vertex u_i of G_i with a vertex v_i of *H* for $1 \le i \le n$. If all the graphs G_i are isomorphic to a graph *G*, then the graph is denoted by $H\,\hat{O}\,G$. Again $H\,\tilde{O}\,(G_1, G_2, \ldots, G_n)$ is a graph obtained by joining a vertex u_i of G_i with a vertex v_i of *H* by an edge for $1 \le i \le n$. If all the graphs G_i are isomorphic to a graph *G*, then the graph is denoted by $H\,\tilde{O}\,G$.

Definition 1.3. Let G be a graph obtained from the path $P_n; x_1x_2 \ldots x_n$ adding a new vertices $y_1y_2 \ldots y_n$ and new edges $y_1x_2, y_nx_{n-1}; x_iy_i$ *for* $1 \le i \le n$ y_ix_{i-1}, y_ix_{i+1} *for* $2 \le i \le n-1$. The family of graphs is called zig-zag triangle.



Definition 1.4[14]. Let *T* be a tree and u_0 and v_0 be the two adjacent vertices in *T*. Let *u* and *v* be the two pendant vertices of *T* such that the length of the path u_0-u is equal to the length of the path v_0-v. If the edge u_0v_0 is deleted from *T* and *u* and *v* are joined by an edge *uv*, then such a transformation of *T* is called an elementary parallel transformation (or an ept) and the edge u_0v_0 is called transformable edge.

If by the sequence of ept's, *T* can be reduced to a path, then *T* is called a T_p -tree (transformed tree) and such sequence regarded as a composition of mappings (ept's) denoted by *P*, is called a parallel transformation of *T*. The path, the image of *T* under *P* is denoted as *P (T).*

A T_P -tree and the sequence of two ept's reducing it to a path are illustrated in Figure 1.2.

a) A T_P –tree T b) An ept $P_1(T)$ c)Second ept $P_2(T)$

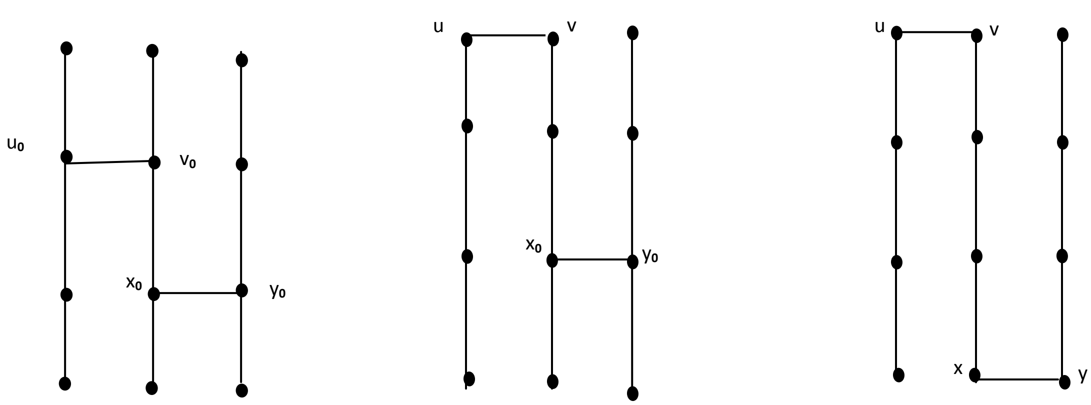


Figure1.2

II MAIN RESULTS

Theorem 2.1. If T be a T_p-tree on m vertices, then the graph $T \hat{O} nC_4$ is a vertex equitable graph.

Proof: Let T be a T_p-tree with m vertices. By the definition of a transformed tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (*i*) $V(P(T)) = V(T)$(ii) $E(P(T)) = (E(T) - E_d) \cup E_p$ where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \ldots, P_k)$ of the epts P used to arrive at the path $P(T)$. Clearly, E_d and E_p have the same number of edges.

Now denote the vertices of $P(T)$ successively by $u_1', u_2', \ldots, u_m'$ starting from one pendant vertex of $P(T)$ right up to the other one. Let $u_{i1}, u_{i2}, \ldots, u_{i(n+1)}$, $v_{i1}, v_{i2}, \ldots, v_{in}$ and $w_{i1}, w_{i2}, \ldots, w_{in}$ $(1 \le i \le m)$ be the vertices of i^{th} copy of P_n with $u_{i(n+1)} = u_i'$.Then $V(T\hat{O}nC_4)$ $= \{u_{ij} : 1 \le i \le m, 1 \le j \le n+1 \ with\ u_{i(n+1)} = u_i'\} \cup \{u_i', v_{ij}, w_{ij} : 1 \le i \le m, 1 \le j \le n\}$ and $E(T\hat{O}nC_4) = E(T) \cup E(nC_4)$. Here $|V(T\hat{O}nC_4)| = m(3n+1)$ and $|E(T\hat{O}nC_4)| = 4mn+m-1$. Let $A = \{0, 1, 2, \ldots, \left\lceil \frac{4mn+m-1}{2} \right\rceil\}$. Define a vertex labeling $f: V(T\hat{O}nC_4) \to A$ as follows.

For $1 \le i \le m$, $1 \le j \le n+1$

$$f(u_{ij}) = \begin{cases} \frac{(4n+1)(i-1)}{2} + 2(j-1) & if\ \ i\ \ is\ odd \\ \frac{(4n+1)i}{2} - 2(j-1) & if\ \ i\ \ is\ even \end{cases}.$$

For $1 \le i \le m$, $1 \le j \le n$

$$f(v_{ij}) = \begin{cases} \frac{(4n+1)(i-1)}{2} + 2j & if\ \ i\ \ is\ odd \\ \frac{(4n+1)i}{2} - 2j & if\ \ i\ \ is\ even \end{cases}, \quad f(w_{ij}) = \begin{cases} \frac{(4n+1)(i-1)}{2} + 2j - 1 & if\ \ i\ \ is\ odd \\ \frac{(4n+1)i}{2} - (2j-1) & if\ \ i\ \ is\ even \end{cases}.$$

Let $v_i v_j$ be a transformed edge in T for some indices i, j, $1 \le i \le j \le m$ .Let P_1 be the ept that deletes the edge $v_i v_j$ and adds an edge $v_{i+t} v_{j-t}$ where t is the distance of v_i from v_{i+t} and the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts.

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Since $v_{i+t}\, v_{j-t}$ is an edge in the path $P(T)$, it follows that $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. Therefore, i and j are of opposite parity, that is, i is odd and j is even or vice-versa.

The induced label of the edge $v_i v_j$ is given by $f^*(v_i\, v_j) = f^*(v_i\, v_{i+2t+1}) = f(v_i) + f(v_{i+2t+1}) = (4n+1)(i+t)$

and $f^*(v_{i+t}\, v_{j-t}) = f^*(v_{i+t}\, v_{i+t+1}) = f(v_{i+t}) + f(v_i\, v_{i+t+1}) = (4n+1)(i+t)$

Therefore, $f^*(v_i\, v_j) = f^*(v_{i+t}\, v_{j-t})$It can be verified that the induced edge labels of $T\,\hat{O}\,nC_4$ are 1, 2, 3,…, $8mn+2m-2$ and $\left|v_f(a) - v_f(b)\right| \le 1$ for all $a, b \in A$. Hence, $T\,\hat{O}\,nC_4$ is a vertex equitable graph.

An example for the vertex equitable labeling of $T\,\hat{O}\,2C_4$ where T is a T_p-tree on *8* vertices is shown in Figure 2.2.

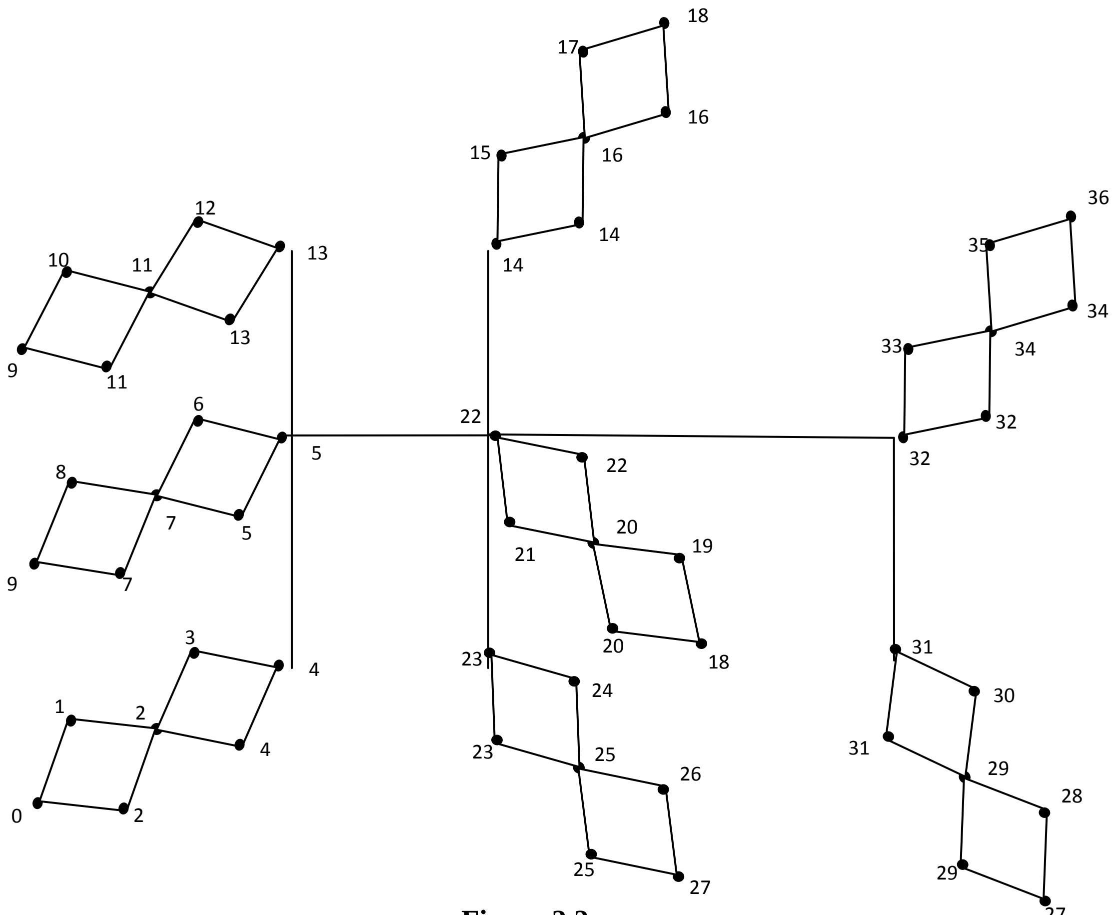

Figure 2.2.

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Theorem 2.2. If G is a zig-zag triangle, then G is a vertex equitable graph.

Proof: Let zig-zag triangles be defined as in definition 1.3. Here $|V(G)|=2n$ and $|E(G)|=4n-3$. Let $A = \{0, 1, 2,..., \left\lceil \frac{4n-3}{2} \right\rceil\}$. Define a vertex labeling $f: V(G) \to A$ as follows. $f(y_1)=1$, $f(x_1)=0$,

$$f(x_{4i-2}) = 8i-5,\ f(y_{4i-2}) = 8i-6 \text{ if } 1 \le i \le \left\lfloor \frac{n+2}{4} \right\rfloor$$

$$f(x_{4i-1}) = 8i-4,\ f(y_{4i-1}) = 8i-3 \text{ if } 1 \le i \le \left\lfloor \frac{n+1}{4} \right\rfloor$$

$$f(x_{4i}) = 8i-2,\ f(y_{4i}) = 8i \text{ if } 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor$$

For $1 \le i \le \left\lfloor \frac{n-1}{4} \right\rfloor$ and $n > 4$ $f(x_{4i+1}) = 8i+1,\ f(y_{4i+1}) = 8i-1$.

It can be verified that the induced edge labels of G are 1, 2,…,4n-3 and $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$. Hence a zig-zag triangle is a vertex equitable graph.

An example for the vertex equitable labeling of zig-zag triangle is shown in Figure 2.3.

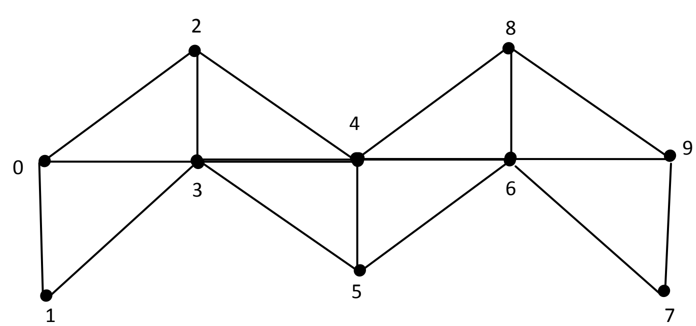

Figure 2.3

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