On decomposition of w-α-$\mathfrak{I}$-continuity

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ABSTRACT

In this paper we will give introduce w-α-$\mathfrak{I}$-continuous mappings and give various characterizations of it.

Key Words and phrases: w-α-$\mathfrak{I}$-open, w-α-$\mathfrak{I}$-continuous.

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I.INTRODUCTION

In [3], Janković and Hamlett introduced the concept of $\mathfrak{I}$-open sets in topological spaces. In [2] Hatir and Noiri introduced the notion of α-$\mathfrak{I}$-open sets and α-$\mathfrak{I}$-continuous functions and in [1] Noiri et al. further investigate the properties of α-$\mathfrak{I}$-continuous functions. The subject of ideals in topological spaces were introduced by Kuratowski[5] and further studied by Vaidyanathaswamy[6]. Corresponding to an ideal a new topology $\tau^*(\mathfrak{I}, \tau)$ called the *-topology was given which is generally finer than the original topology having the kuratowski closure operator $cl^*(A) = A \cup A^*(\mathfrak{I},\tau)$[7], where $A^*(\mathfrak{I},\tau) = \{x \in X : U \cap A \notin \mathfrak{I}$ for every open subset U of x in X called a local function of A with respect to $\mathfrak{I}$ and τ. We will write τ^* for $\tau^*(\mathfrak{I}, \tau)$.

The following section contains some definitions and results that will be used in our further sections.

Definition 1.1.[5]: Let (X, τ) be a topological space. An ideal $\mathfrak{I}$ on X is a collection of non-empty subsets of X such that (a) $\phi \in \mathfrak{I}$ (b) $A \in \mathfrak{I}$ and $B \in \mathfrak{I}$ implies $A \cup B \in \mathfrak{I}$ (c) $B \in \mathfrak{I}$ and $A \subset B$ implies $A \in \mathfrak{I}$.

Definition 1.2 [2] : Let$(X,\tau,\mathfrak{I})$ be an ideal space and A be any subset of X. Then A is said to be α-$\mathfrak{I}$-open if

$A \subset int(cl^*(int(A)))$.

Definition 1.3 [2]: Let $(X,\tau,\mathfrak{I})$ and (Y,σ) be two topological spaces. Then a map $f : (X,\tau,\mathfrak{I}) \to (Y,\sigma)$ is said to be α-$\mathfrak{I}$-continuous if inverse image of every open set in Y is α-$\mathfrak{I}$-open in X.

Definition 1.4.[4] : A mapping $f : (X,\tau,\mathfrak{I}) \to (Y,\sigma)$ is said to be pointwise $\mathfrak{I}$-continuous if the inverse image of every open set in Y is τ^*-open in X.

Lemma 1.5.[3] : Let $(X,\tau,\mathfrak{I})$ be an ideal space and Y be subset of X. Then

$\mathfrak{I}_Y = \{I \cap Y \mid I \in \ \}$ is an ideal on Y.

II.RESULTS

Definition 2.1: Let $(X,\tau,\mathfrak{I})$ be an ideal space and A be any subset of X. Then A is said to be w-α*-$\mathfrak{I}$-open if

$A \subset int^*(cl(int^*(A))$.

Definition 2.2: Let $(X,\tau,\mathfrak{I})$ and (Y,σ) be two topological spaces. Then a map $f:(X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma)$ is said to be w-α-$\mathfrak{I}$-continuous if inverse image of every open set in Y is w- α *-$\mathfrak{I}$-open in X.

i.e. f is w-α-$\mathfrak{I}$-continuous if $\forall$ V ϵ σ, $f^{-1}(V)$ is w-α*-$\mathfrak{I}$-open subset of X.

Definition 2.3: Let $(X,\tau,\mathfrak{I})$ and $(Y,\sigma, \mathcal{J})$ with $\mathcal{J}=f(\mathfrak{I})$ be two topological spaces. Then a map

$f:(X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma, \mathcal{J})$ is said to be w*-α-$\mathfrak{I}$-continuous if inverse image of every σ*-open set in Y is w-α*-$\mathfrak{I}$-open in X.

i.e. f is w*-α-$\mathfrak{I}$-continuous if $\forall$ V ϵ σ^*, $f^{-1}(V)$ is w-α*-$\mathfrak{I}$-open subset of X.

Remark 2.4: Since $\sigma \subset \sigma^*$. Therefore, it can be easily seen that every w*-α-$\mathfrak{I}$-continuous map is w-α-$\mathfrak{I}$-continuous.

Definition 2.5: Let $(X,\tau,\mathfrak{I})$ and $(Y,\sigma,\mathcal{J})$ with $\mathcal{J}=f(\mathfrak{I})$ be two ideal topological spaces. Then a map

$f:(X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma,\mathcal{J})$ is said to be w-α-$\mathfrak{I}$-irresolute if inverse image of every w-α*-$\mathcal{J}$-open subset in Y is w-α*-$\mathfrak{I}$-open in X.

i.e. f is w-α-$\mathfrak{I}$-irresolute if $\forall$ w-α*-$\mathcal{J}$-open subset V in Y, $f^{-1}(V)$ is w-α*-$\mathfrak{I}$-open subset of X.

Remark 2.6: Since, every τ*-open subset V in an ideal space $(X,\tau,\mathfrak{I})$ is w-α*-$\mathfrak{I}$-open and every open subset of X is τ*-open. Therefore,

1.) Every pointwise-$\mathfrak{I}$-continuous map is w-α-$\mathfrak{I}$-continuous.
2.) Every w-α-$\mathfrak{I}$-irresolute is w-α-$\mathfrak{I}$-continuous map.

Theorem 2.7: Let $(X,\tau,\mathfrak{I})$ and (Y,σ) be two topological spaces and $f:(X,\tau,\mathfrak{I}) \rightarrow (Y,\sigma)$ be any map. Then prove that the following are equivalent:

a) f is w-α-$\mathfrak{I}$-continuous.
b) For each x ϵ X and any open set V containing f(x) in Y, there exists a w-α-$\mathfrak{I}$-open subset U of x such that $f(U) \subset V$.
c) The inverse image of every closed set in Y is w-α-$\mathfrak{I}$-closed in X.

Proof: (a) ⇒(b): Let x ϵ X be any element and V be any open set in Y containing f(x). Then by (1), $f^{-1}(V)$ is w-α*-𝔗-open subset of X containing x. Let U = $f^{-1}(V)$. Hence there exist w-α*-𝔗-open subset U of X containing x such that f(U) = $f(f^{-1}(V)) \subset V$.

(b)⇒(a): Let V be an open subset of Y. Then there can be two possibilities:

1.) $f^{-1}(V) = \phi$, then we have nothing to prove.

2.) $f^{-1}(V) \neq \phi$. Let x ϵ $f^{-1}(V)$ then f(x) ϵ V. Now by (2), there exist w-α-𝔗-open subset U of X containing x such that f(x) ϵ f(U) ⊂ V and so x ϵ $f^{-1}(f(U)) \subset f^{-1}(V)$. Therefore, x ϵ U ⊂ $f^{-1}(f(U)) \subset f^{-1}(V)$. Hence ∀ x ϵ $f^{-1}(V)$, there exist w-α*-𝔗-open subset U of X containing x such that x ϵ U ⊂ $f^{-1}(V)$. This implies that $f^{-1}(V)$ is the union of w-α*-𝔗-open sets. But union of w-α*-𝔗-open sets is also w-α*-𝔗-open set. So, $f^{-1}(V)$ is w-α*-𝔗-open subset of X. Hence f is w-α-𝔗-continuous.

(a)⇔(c): f is w-α-𝔗-continuous if and only if inverse image of every open subset V in Y is w-α*-𝔗-open in X i.e. if and only if ∀ V ϵ σ, $f^{-1}(V)$ is w-α*-𝔗-open subset of X if and only if for every closed set F in Y i.e. Y-F is open in Y, $f^{-1}(Y\text{-}F)$ is w-α*-𝔗-open in X i.e. if and only if for every closed F in Y, X- $f^{-1}(F)$ is w-α*-𝔗-open in X i.e. if and only if for every closed F in Y, $f^{-1}(F)$ is w-α*-𝔗-closed in X if and only if inverse image of every closed set in Y is w-α*-𝔗-closed in X.

Theorem 2.8: Let f : (X,τ,𝔗) → (Y,σ) be any w-α-𝔗-continuous map. If G be any τ-open subset of X then prove that

f|G : (G, τ|G, 𝔗|G) → (Y,σ) is w-α-𝔗-continuous.

Proof: Let W be any open subset of Y. Then f is w-α-𝔗-continuous implies that $f^{-1}(W)$ is w-α*-𝔗-open in X.

Now, G is τ-open subset of X implies that $f^{-1}(W) \cap G \subset int^*(cl(int^*(f^{-1}(W)))) \cap int(G)$

$$= int^*(cl(int^*(f^{-1}(W)))) \cap int^*(int(G))$$

$$= int^*(cl(int^*(f^{-1}(W))) \cap int(G))$$

$$\subset int^*(cl(int^*(f^{-1}(W) \cap G))).$$

Therefore, $f^{-1}(W) \cap G$ is w-α*-𝔗-open subset of G.

Hence f|G is w-α-𝔗-continuous.

Theorem 2.9: Let f : (X,τ,𝔗) → (Y,σ,𝒥) and g : (Y,σ,𝒥) → (Z,η) be any two mappings. Then prove that the following hold:

(a) gof is w-α-𝔗-continuous if g is continuous and f is w-α-𝔗-continuous.

(b) gof is w-α-$\mathfrak{T}$-continuous if g is w-α-$\mathfrak{T}$-continuous and f is w-α-$\mathfrak{T}$-irresolute.

Proof: (a): Let W be any open subset of Z. Then g is continuous implies that $g^{-1}(W)$ is open in Y. Further, f is w-α-$\mathfrak{T}$-continuous implies that $f^{-1}(g^{-1}(W))$ is w-α*-$\mathfrak{T}$-open subset of X and so $(gof)^{-1}(W)$ is w-α-$\mathfrak{T}$-open subset of X.

Hence gof is w-α-$\mathfrak{T}$-continuous.

(b): Let W be any open subset of Z. Then g is w-α-$\mathfrak{T}$-continuous implies that $g^{-1}(W)$ is w-α*-$\mathcal{J}$-open in Y. Further, f is w-α-$\mathfrak{T}$-irresolute implies that $f^{-1}(g^{-1}(W))$ is w-α*-$\mathfrak{T}$-open subset of X and so $(gof)^{-1}(W)$ is w-α-$\mathfrak{T}$-open subset of X.

Hence gof is w-α-$\mathfrak{T}$-continuous .

Theorem 2.10 : Let $f : (X,\tau,\mathfrak{T}) \rightarrow (Y,\sigma)$ be any map. Then prove that the following are equivalent:

a) f is w-α - $\mathfrak{T}$ -continuous.
b) The inverse image of each closed set in Y is w-α*- $\mathfrak{T}$-closed in X.
c) $cl^*(int(cl^*(f^{-1}(B)))) \subset f^{-1}(cl(B))$ for each subset B of Y.
d) $f(cl^*(int(cl^*(A)))) \subset cl(f(A))$ for each subset A of X.

Proof: (a)⇔(b) follows from the above Theorem 2.7.

We will prove that (b)⇒(c). Let B be any subset of Y then cl(B) is closed subset of Y. So by (b), $f^{-1}(cl(B))$ is w-α*- $\mathfrak{T}$-closed subset of X and so X- $f^{-1}(cl(B))$ is w-α*- $\mathfrak{T}$-open subset of X.

Thus
$$X- f^{-1}(cl(B)) \subset int^*(cl(int^*(X- f^{-1}(cl(B))))$$
$$= int^*(cl(X-cl^*(f^{-1}(cl(B)))))$$
$$= int^*(X-int(cl^*(f^{-1}(cl(B)))))$$
$$= X- cl^*(int(cl^*(f^{-1}(cl(B))))).$$

Therefore, $$cl^*(int(cl^*(f^{-1}(cl(B))))) \subset f^{-1}(cl(B)).$$

Hence $$cl^*(int(cl^*(f^{-1}(B)))) \subset f^{-1}(cl(B))$$

(c)⇒(d): Let A be any subset of X then f(A) be subset of Y. So by (c), for B = f(A)

$$cl^*(int(cl^*(f^{-1}(f(A))))) \subset f^{-1}(cl(f(A)))$$

and so $cl^*(int(cl^*(A))) \subset f^{-1}(cl(f(A)))$, since $A \subset f^{-1}(f(A))$ for any subset A of X.

and so $f(cl^*(int(cl^*(A)))) \subset f(f^{-1}(cl(f(A))))$.

Therefore, $f(cl^*(int(cl^*(A)))) \subset cl(f(A))$, since $f(f^{-1}(B)) \subset B$ for any subset B of Y.

Hence $f(cl^*(int(cl^*(A)))) \subset cl(f(A))$ for any subset A of X.

(d)⇒(a): Let W be any open subset of Y then $f^{-1}(Y-W)$ is any subset of X.

So by (d), $f(cl^*(int(cl^*(f^{-1}(Y-W))))) \subset cl(f(f^{-1}(Y-W)))$

And so $f(cl^*(int(cl^*(X-f^{-1}(W))))) \subset cl(Y-W)$, since $f^{-1}(Y-W) = X-f^{-1}(W)$ and $f(f^{-1}(B) \subset B$ for any subset B of Y.

But Y-W is closed subset of Y. So cl(Y-W) = Y-W.

implies that $f(cl^*(int(cl^*(X-f^{-1}(W))))) \subset Y-W$.

And so $f^{-1}(f(cl^*(int(cl^*(X-f^{-1}(W)))))) \subset f^{-1}(Y-W) = X-f^{-1}(W)$.

Therefore, $f^{-1}(W) \subset X- cl^*(int(cl^*(X-f^{-1}(W))))$

And so $f^{-1}(W) \subset int^*(cl(int^*(f^{-1}(W))))$.

Hence $f^{-1}(W)$ is w-α*- $\mathfrak{T}$-open subset of X.

Hence f is w-α- $\mathfrak{T}$-continuous.

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